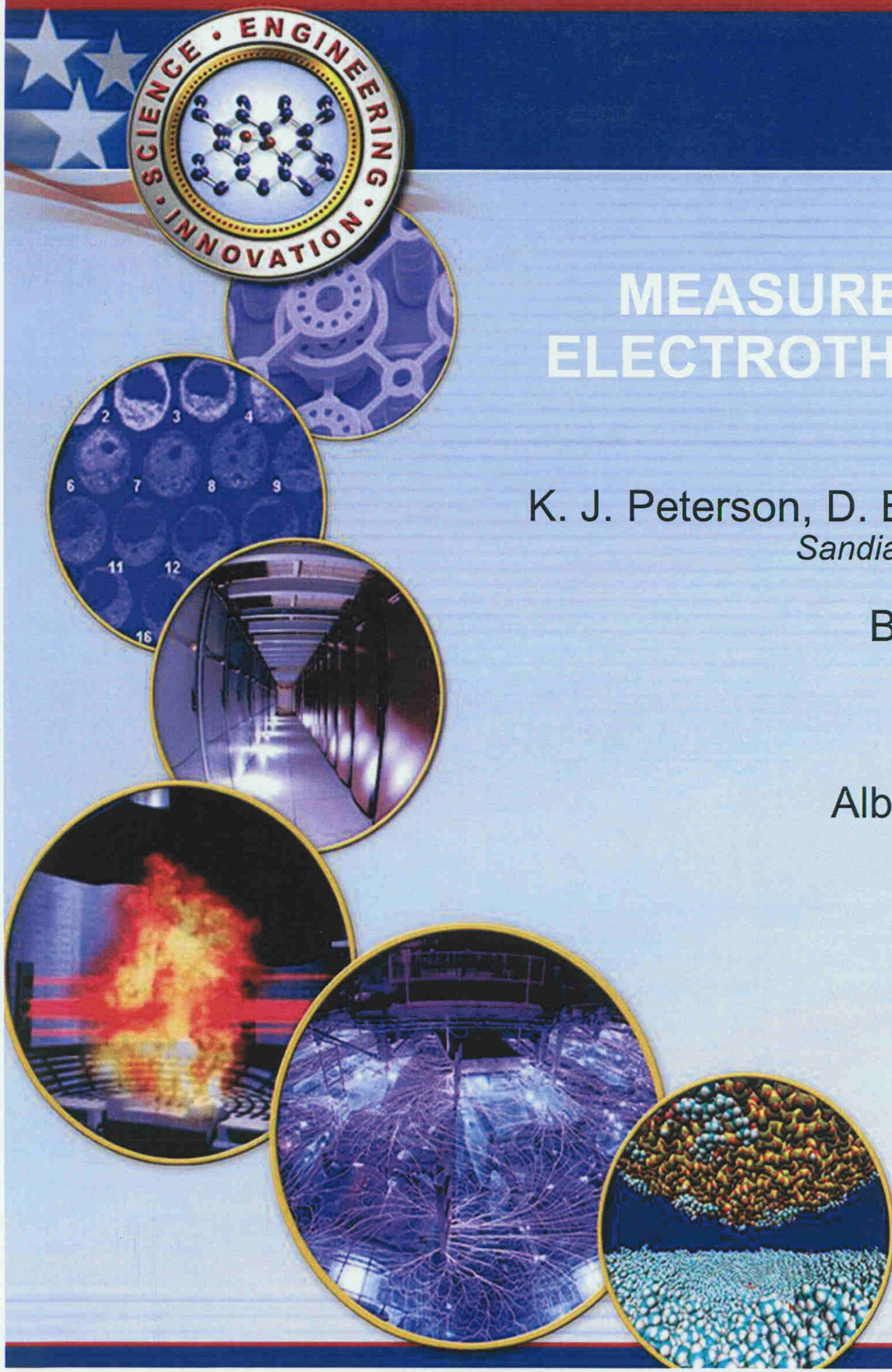

MEASUREMENT AND SIMULATION OF ELECTROTHERMAL INSTABILITY GROWTH

K. J. Peterson, D. B Sinars, E. P Yu, S. A. Slutz, M. C. Herrmann
Sandia National Laboratories, Albuquerque, NM

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General Atomics, San Diego, CA

MagLIF Workshop
Albuquerque, NM, Feb. 5-8, 2012

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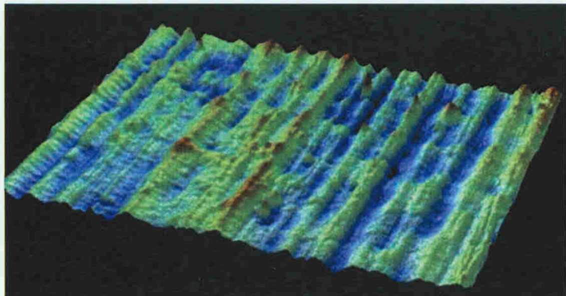
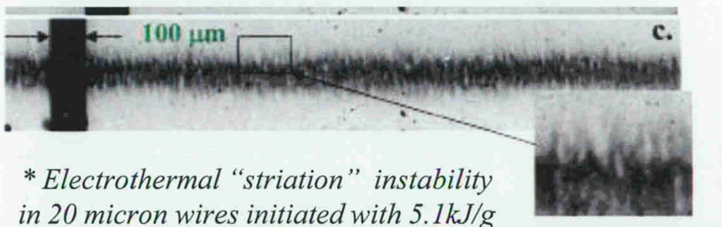
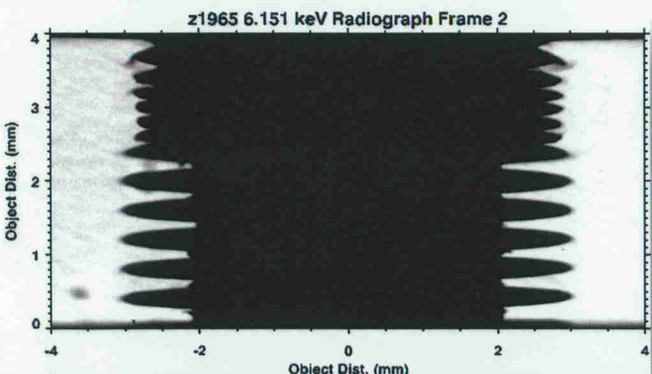
Accurately predicting instability growth and mitigation strategies is dependent on validating our computational models





* Electrothermal "striation" instability in 20 micron wires initiated with 5.1kJ/g



Initial Surface Roughness

↓

Initiation Phase
Electrothermal Instabilities

* *A.G. Rousskikh et al., Physics of Plasmas (2008).*

↓

Implosion Phase
Magneto Rayleigh Taylor (MRT) Instabilities

* *D. B. Sinars et al.*

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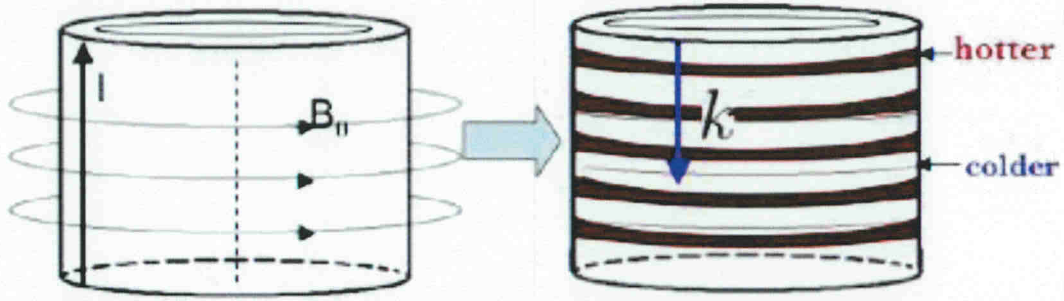
Electrothermal instabilities occur when material conductivity is dependent on temperature

Striations

$$\frac{d\eta(T)}{dT} > 0$$

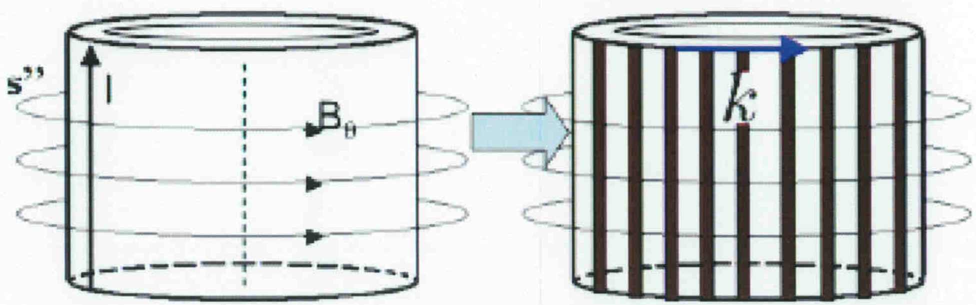
(Also sometimes referred to as thermal overheat Instabilities)



which couple to Rayleigh-Taylor

Filamentations

$$\frac{d\eta(T)}{dT} < 0$$



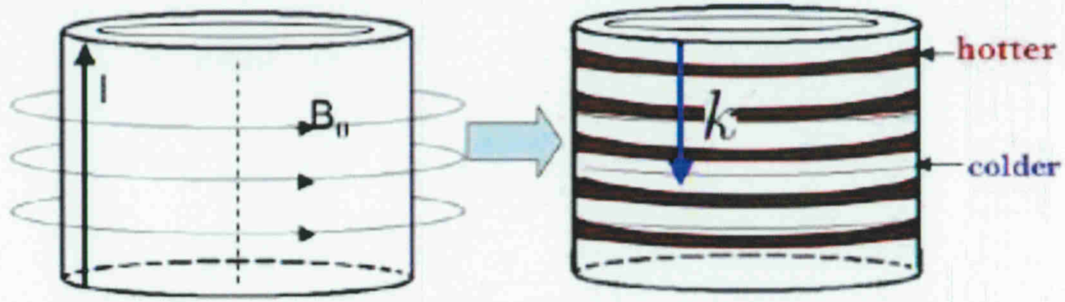
This is commonly the situation that occurs when the term "electrothermal instabilities" is referred to in the literature

which can tranform a 2D (r,z) problem into a fully 3D configuration

Electrothermal instabilities occur when material conductivity is dependent on temperature

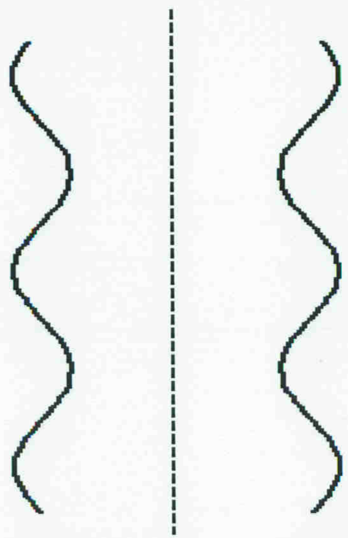
$$\frac{d\eta(T)}{dT} > 0$$

Striations



which couple to Rayleigh-Taylor

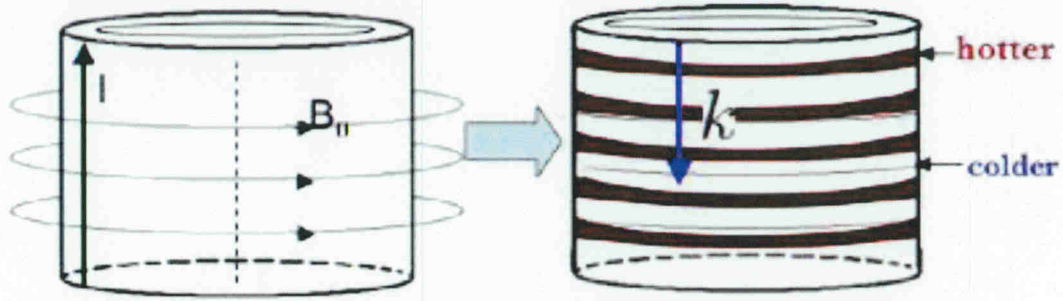
Consider a small surface perturbation on a cylindrical liner



Electrothermal instabilities occur when material conductivity is dependent on temperature

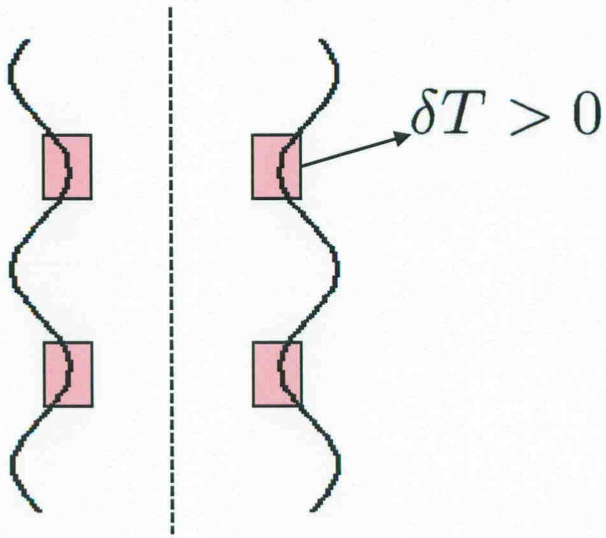
$$\frac{d\eta(T)}{dT} > 0$$

Striations



which couple to Rayleigh-Taylor

Consider a small surface perturbation on a cylindrical liner

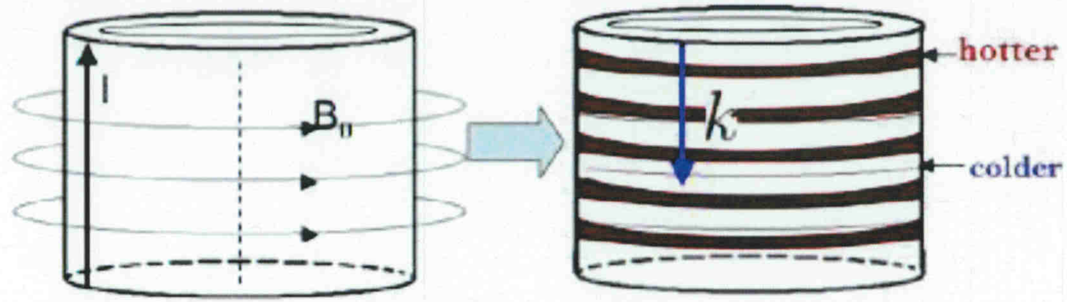


Since $B_\theta \sim I/r$, the current density is enhanced which increases localized ohmic heating, $\eta\, j^2$.

Electrothermal instabilities occur when material conductivity is dependent on temperature

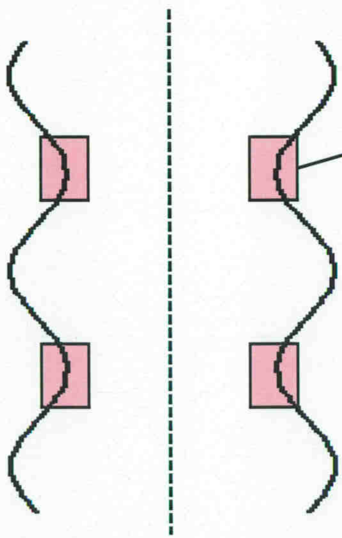
$$\frac{d\eta(T)}{dT} > 0$$

Striations



which couple to Rayleigh-Taylor

Consider a small surface perturbation on a cylindrical liner



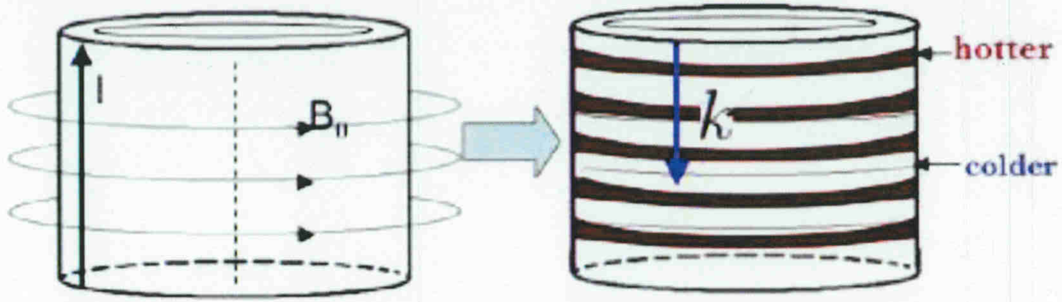
Since $B_\theta \sim I/r$, the current density is enhanced which increases localized ohmic heating, $\eta\, j^2$.

Then, η increases which consequently further enhances the localized ohmic heating,

Electrothermal instabilities occur when material conductivity is dependent on temperature

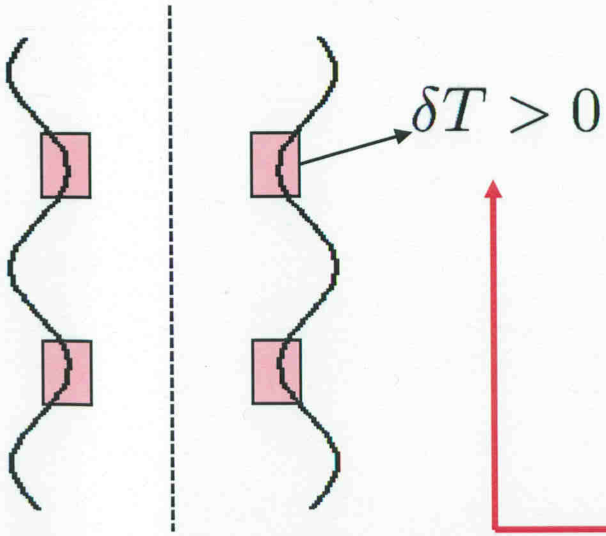
$$\frac{d\eta(T)}{dT} > 0$$

Striations



which couple to Rayleigh-Taylor

Consider a small surface perturbation on a cylindrical liner



Since $B_\theta \sim I/r$, the current density is enhanced which increases localized ohmic heating, $\eta\ j^2$.

Then, η increases which consequently further enhances the localized ohmic heating,

which leads to increased δT

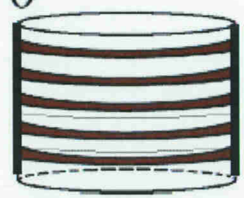
Electrothermal growth rates can be estimated from the equation for thermal balance

$$\rho c_V \frac{\partial T}{\partial t} = \overset{\text{thermal conduction}}{\cancel{\nabla \cdot (\kappa \nabla T)}} - \overset{\text{radiation}}{\cancel{q_r}} + \overset{\text{Ohmic heating}}{\eta j^2} - \overset{\text{pdV}}{\cancel{p \nabla \cdot \mathbf{v}}}$$

Perturbing this equation produces the growth rate

$$\Gamma = \frac{(d\eta/dT) j^2}{\rho c_V} \left[1 - \cancel{\frac{2\cos^2\alpha}{1 + \Gamma/\Gamma_0}} \right]$$

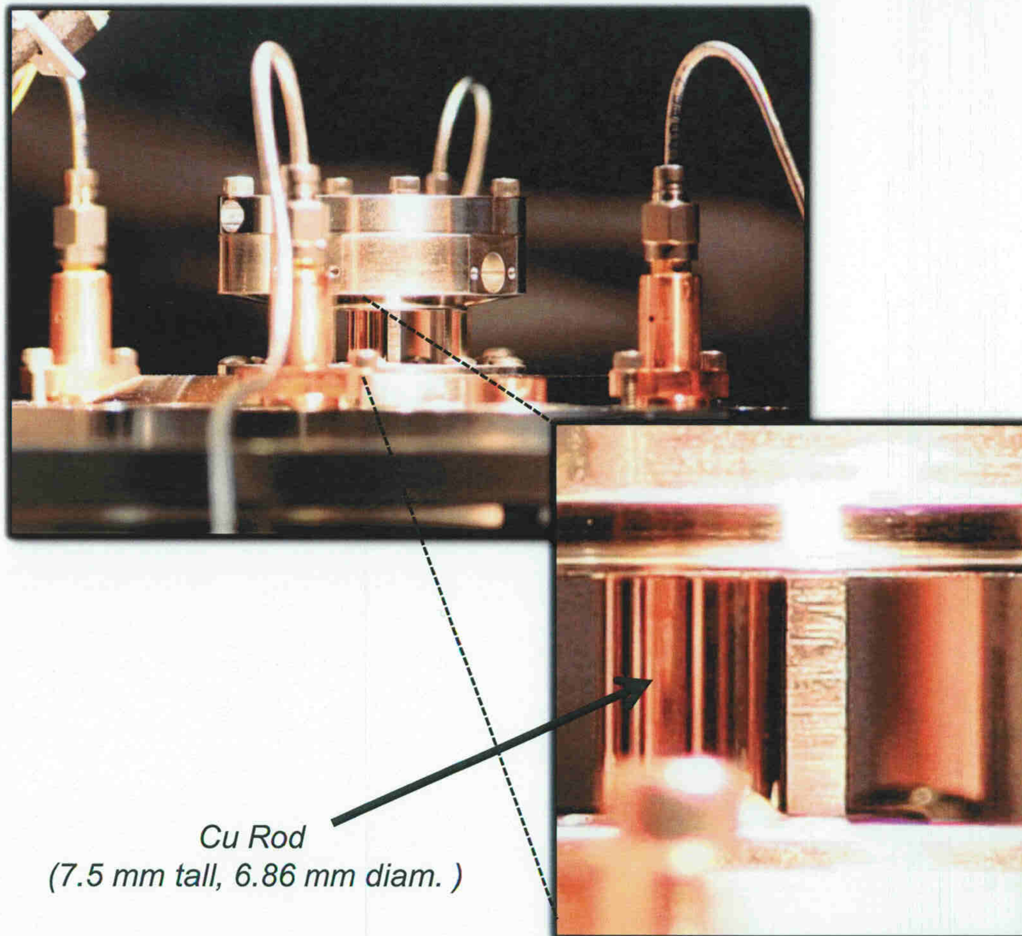
fastest growing modes will have $\cos\alpha = 0$ i.e. striations



Experiments were performed to evaluate initial perturbation growth from surface roughness and test code predictions

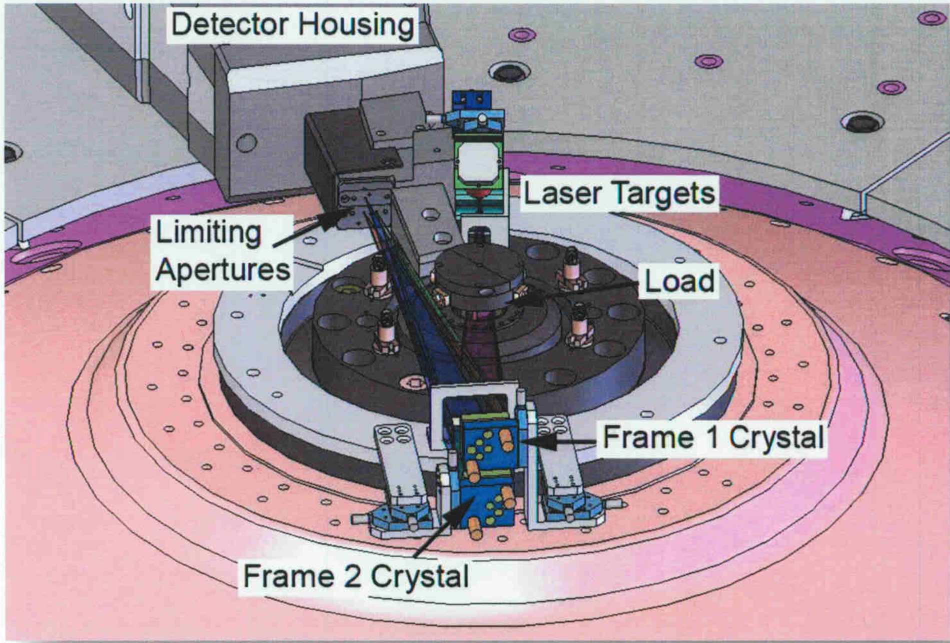
- Solid Cu and Al rods (liner AR=1.0) were driven with ~20MA of current in ~100ns using Sandia's Z accelerator
- Solid rods were used to limit convergence and MRT growth
- Rods were fabricated and characterized by General Atomics
  - Cu, ~5-10nm RMS surface finish (diamond turned)
  - Al (5052), ~10-30nm RMS surface finish (diamond turned)

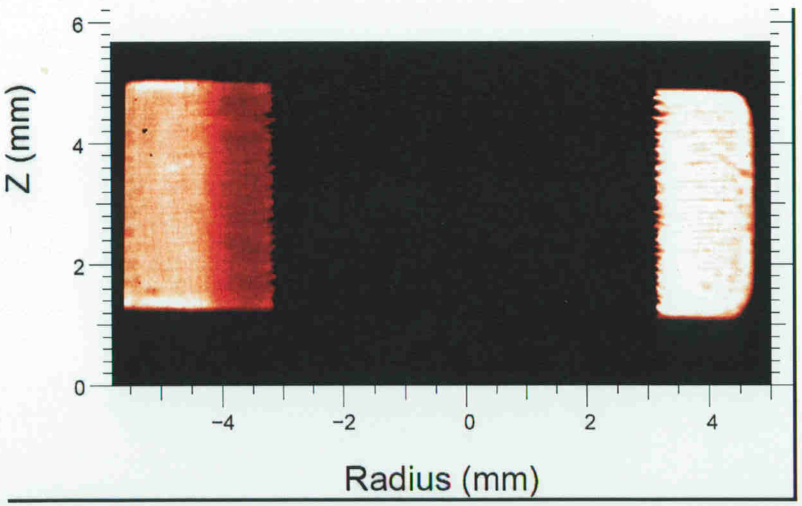
Shot	Material	Diameter	Radiography
1800	Al	12 mm	None
1801	Al	6.86	(1) 6151
1802	Cu	6.86	(2) 6151
1913	Al	6.86	(2) 6151
1916	Cu	6.86	(2) 6151
1922	Cu	6.86	(1) 6151 (1) 1865



2-frame 6.151 keV monochromatic crystal backlighting was used to image instability growth

- Adjustable interframe time (2-20 ns)
  - » Most experiments used 14 ns due to the expected slow evolution of instabilities
- Preshot radiographs were taken to accurately measure image magnification and significantly increase precision
- One experiment used a new two color capability (1.865 keV frame and 6. 151 keV frame taken 2 ns apart)





2-frame 6.151 keV Crystal Imaging

- *Monochromatic (~0.5 eV bandpass)*
- *~15 micron resolution*
- *Large FOV ~ 4x10 mm*

D.B. Sinars et al., Rev. Sci. Instrum. 75, 3672 (2004).
G.R. Bennett et al., RSI (2008).

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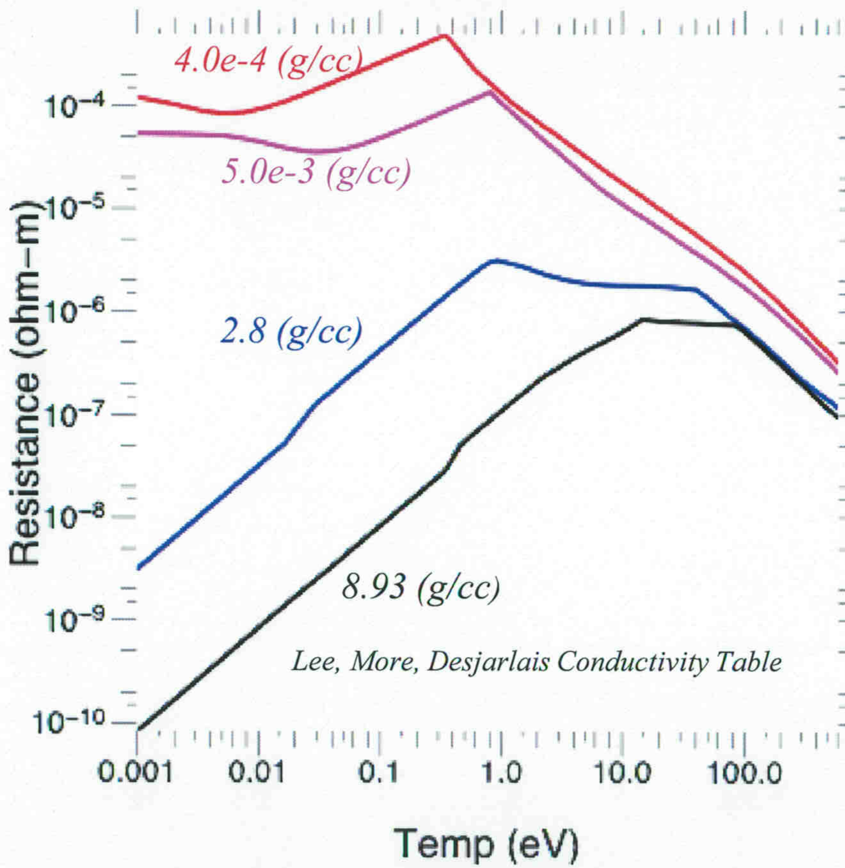
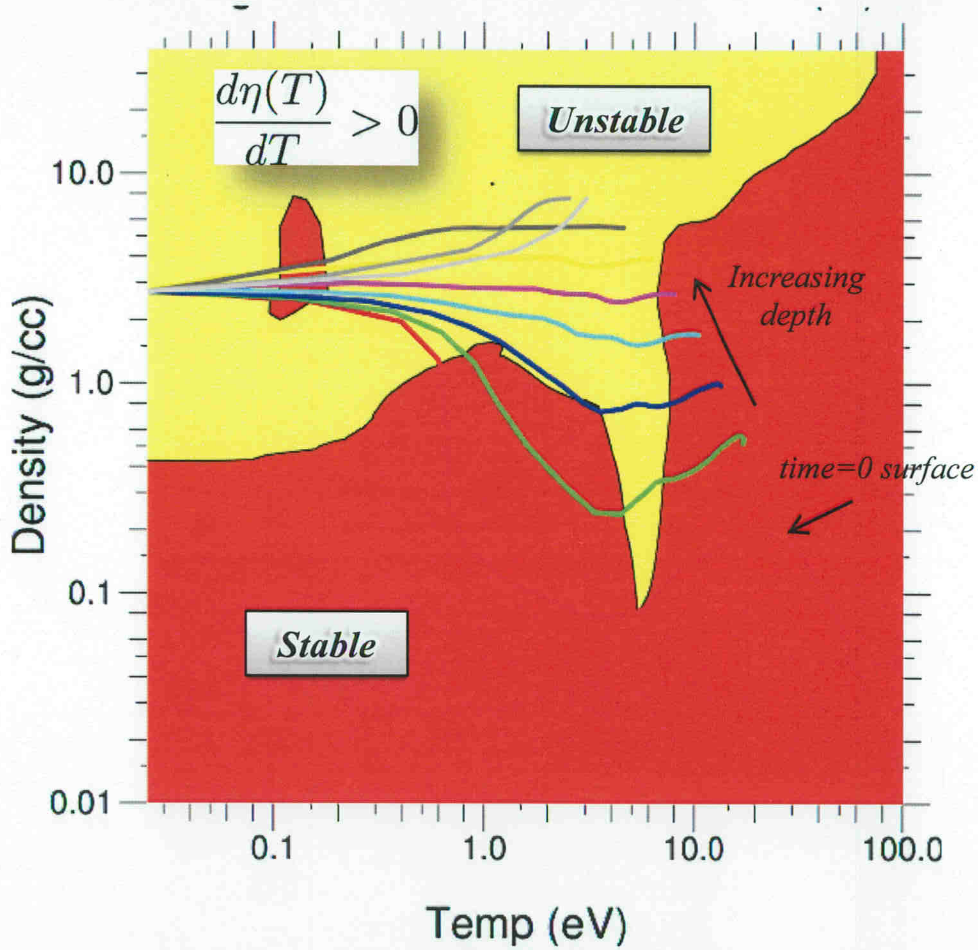
Significant portions of the rod are unstable to the "striation" electrothermal instability

Cu Resistivity

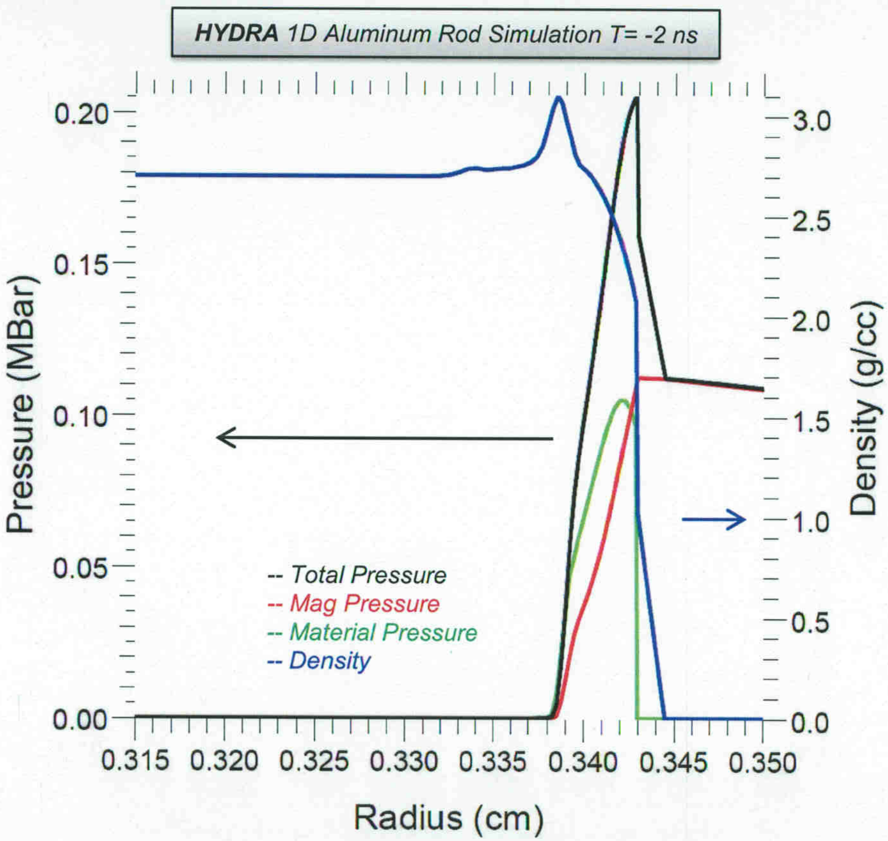


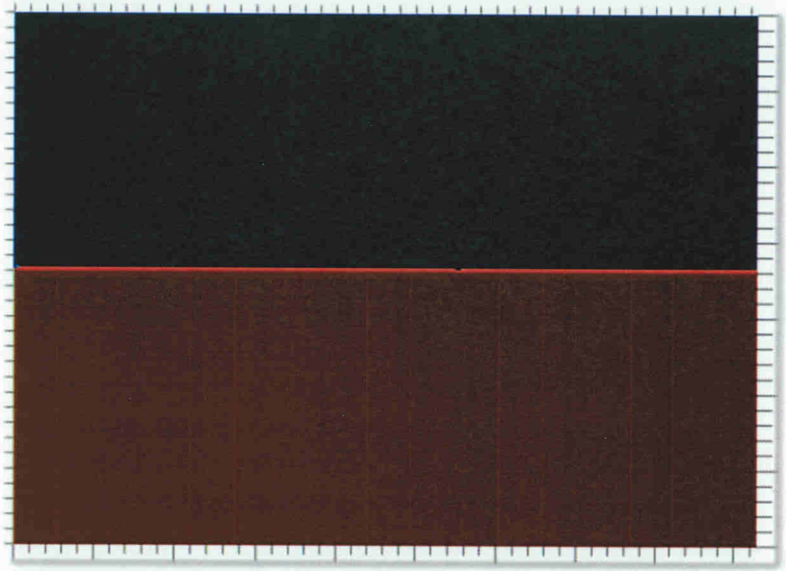
Small changes in density lead to enormous changes in resistivity at low temperatures!

Predicted Lagrangian Phase Space of Al Rod



As soon as current begins to flow a nonlinear magnetic diffusion wave propagates into the rod

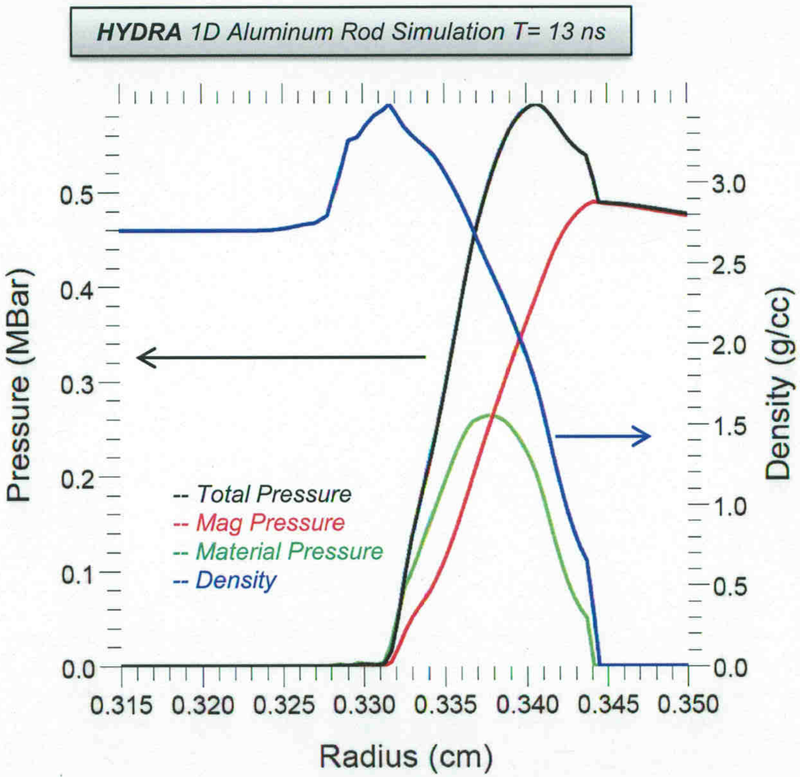


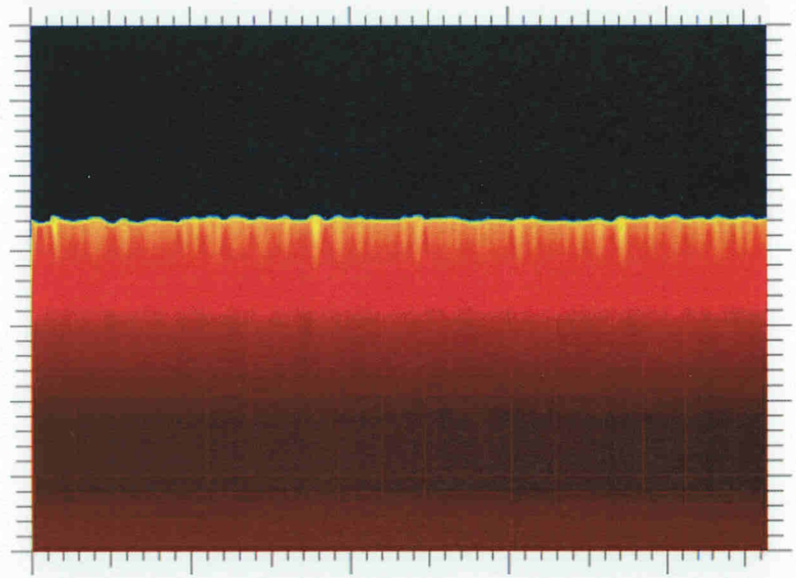


-In the outer surface layers, a positive total pressure gradient exists which causes the surface material to expand

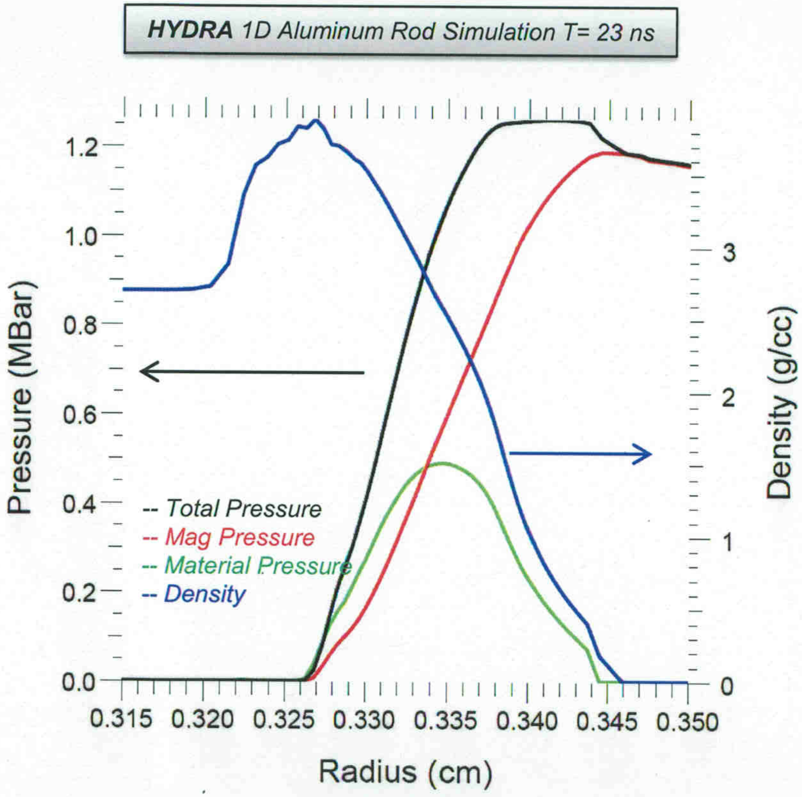
*- Since the density gradient has the same sign as the total pressure gradient in this region, which Includes what we define to be the interface (0.1*density(peak)), it is MRT* ***stable***

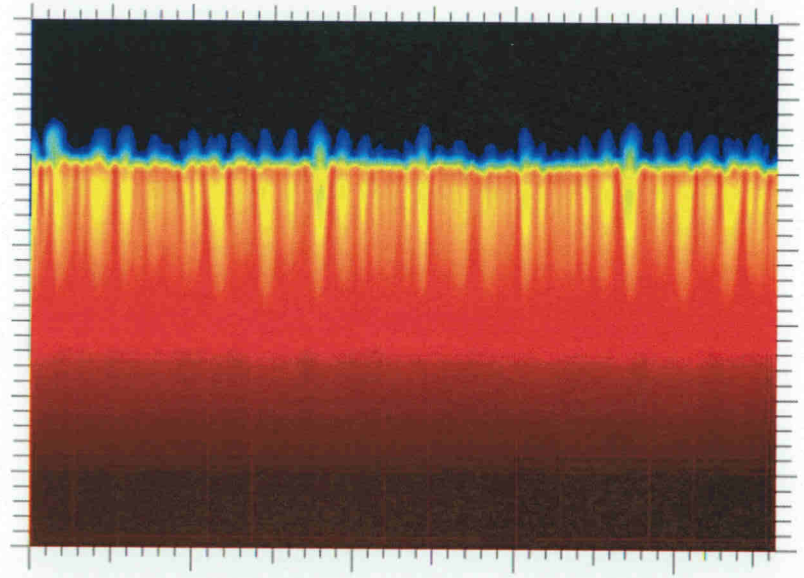
Significant instability growth is observed in regions stable to MRT as the surface of the rod expands





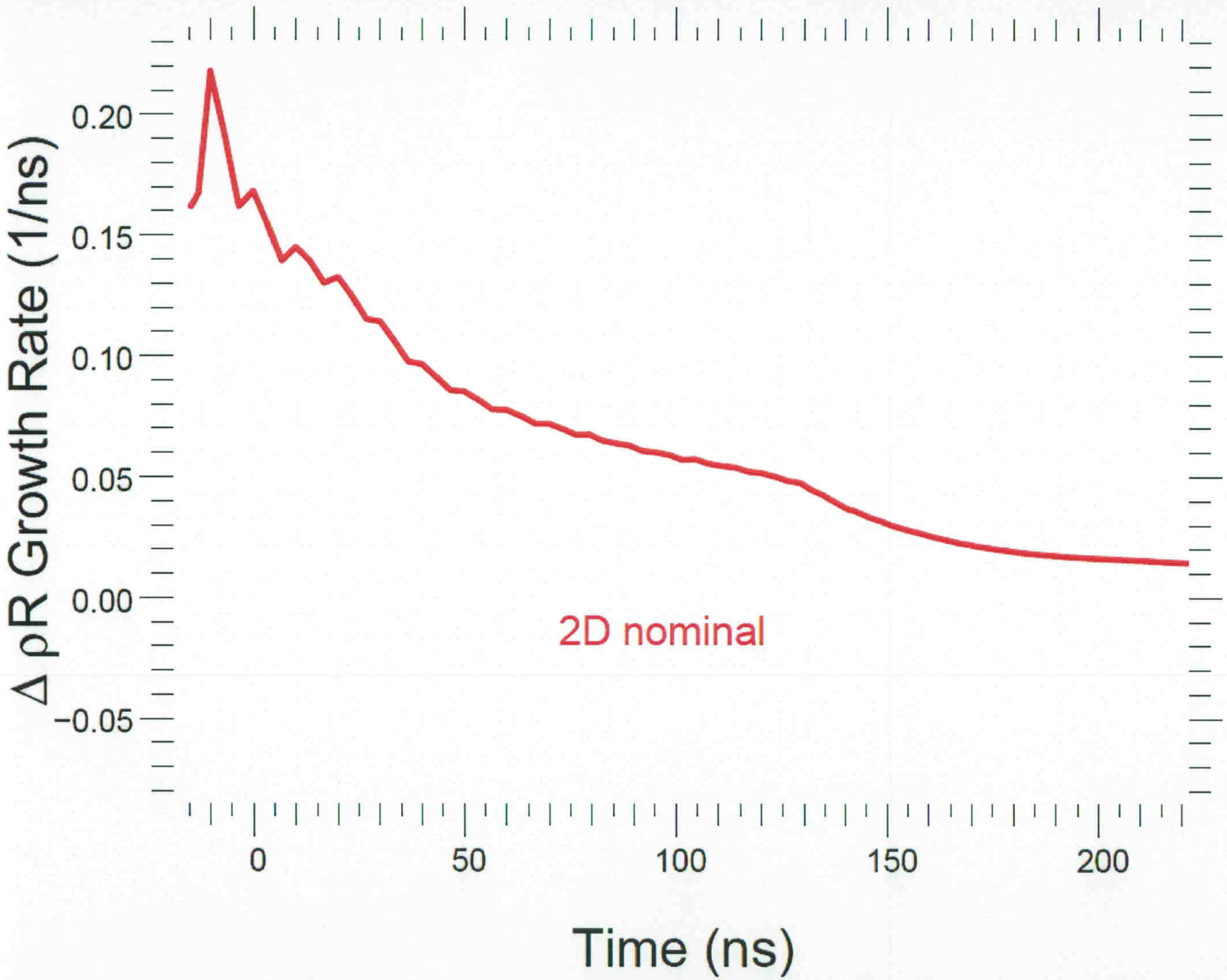
Rod is unstable to MRT growth as it begins to compress under the magnetic pressure





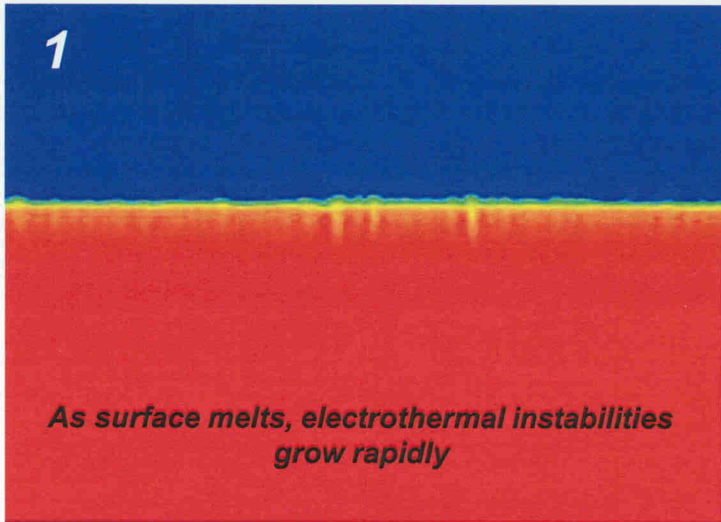
- Density gradient now has the opposite sign as the total pressure gradient in the outer surface layers and is consequently MRT ***unstable***

The fastest growth occurs immediately after current begins to rise



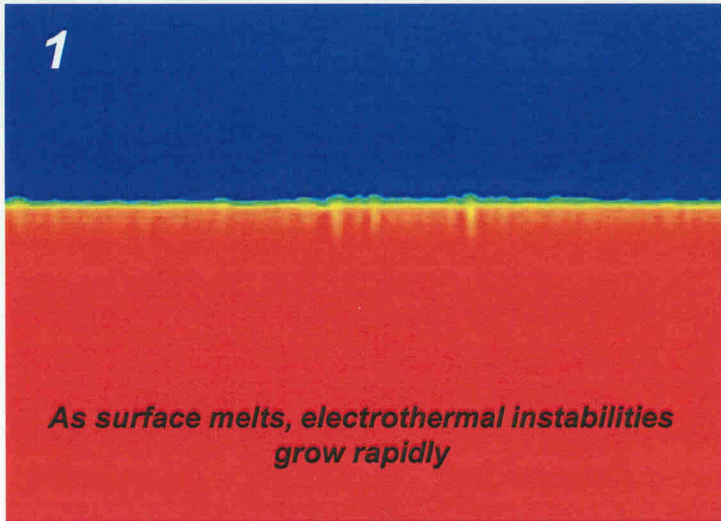
2D simulations show electro-thermal instabilities develop after melt and seed later MRT growth

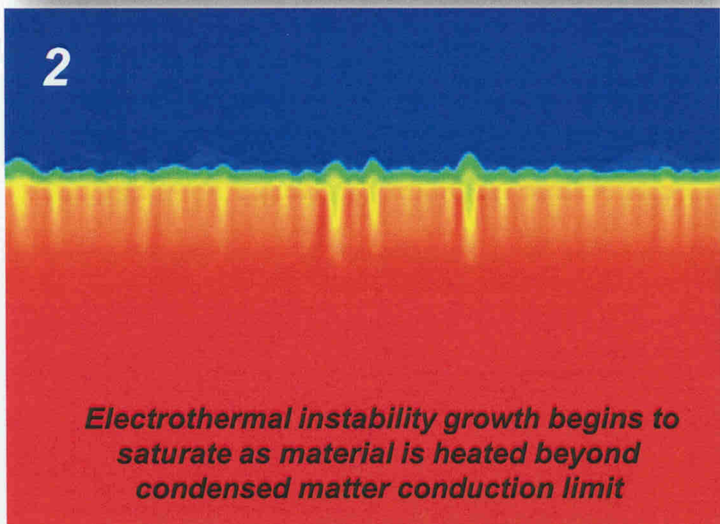
Aluminum Rod



2D simulations show electro-thermal instabilities develop after melt and seed later MRT growth

Aluminum Rod





2D simulations show electro-thermal instabilities develop after melt and seed later MRT growth

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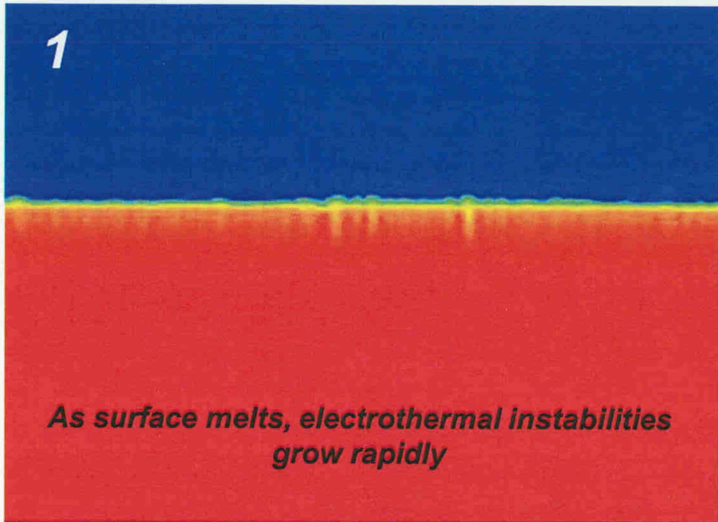
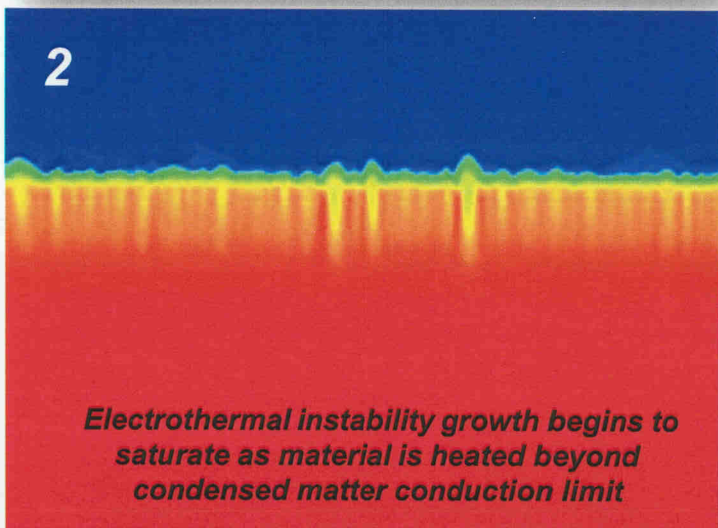
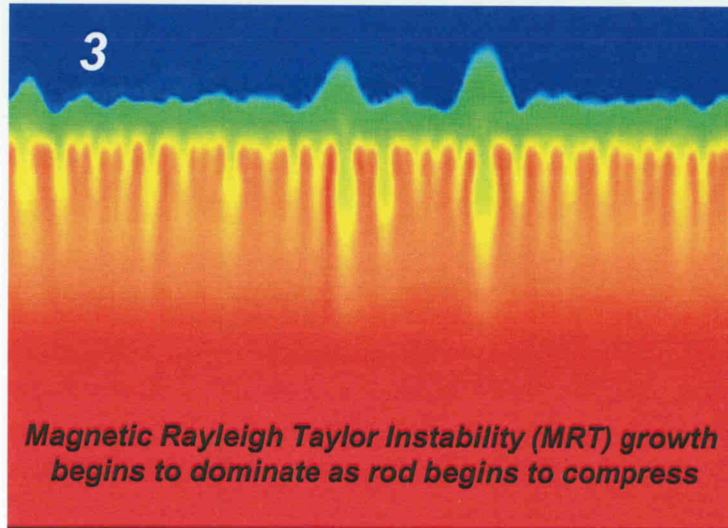
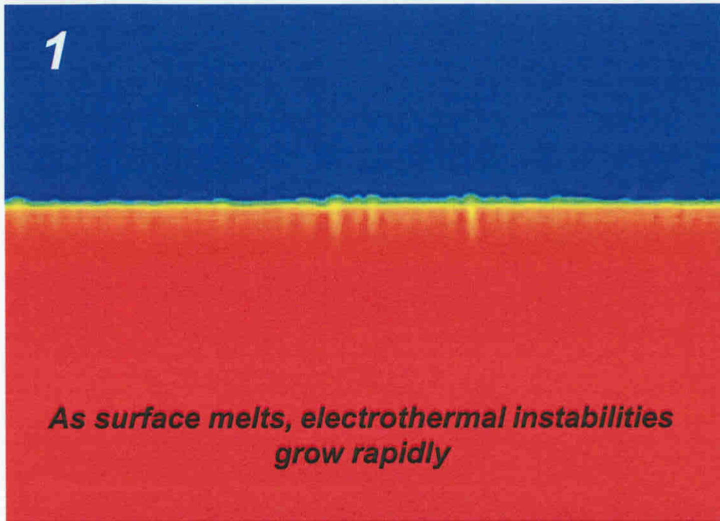


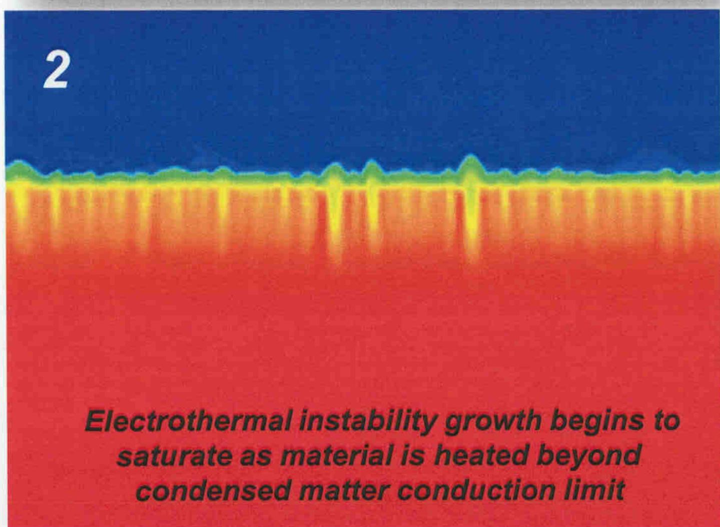


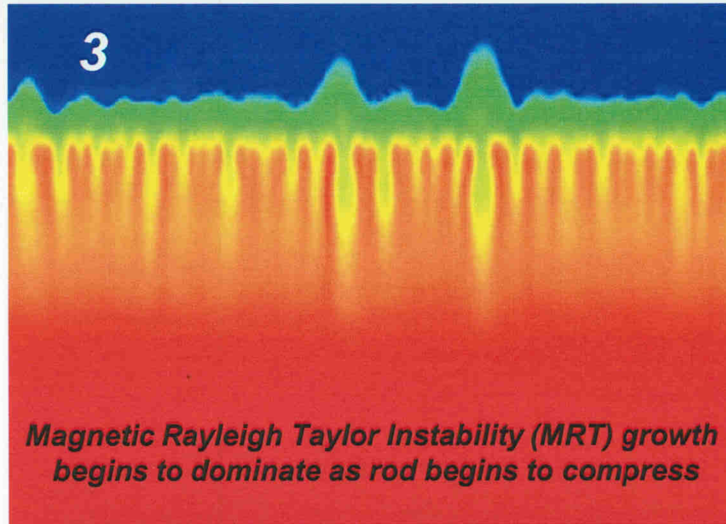


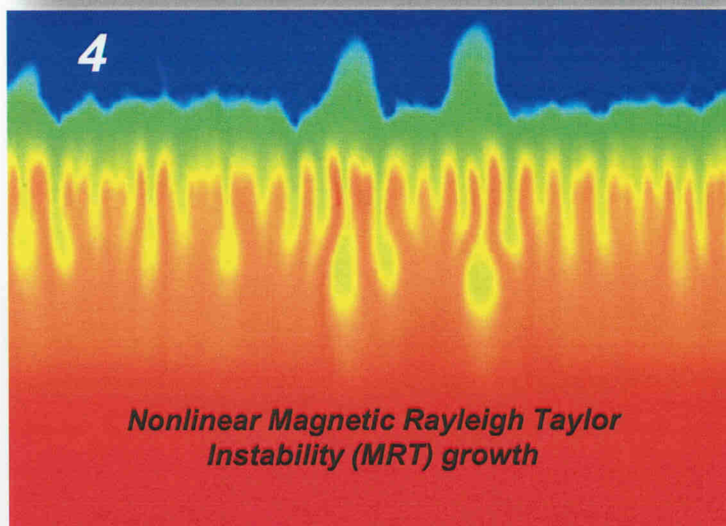
2D simulations show electro-thermal instabilities develop after melt and seed later MRT growth

Aluminum Rod





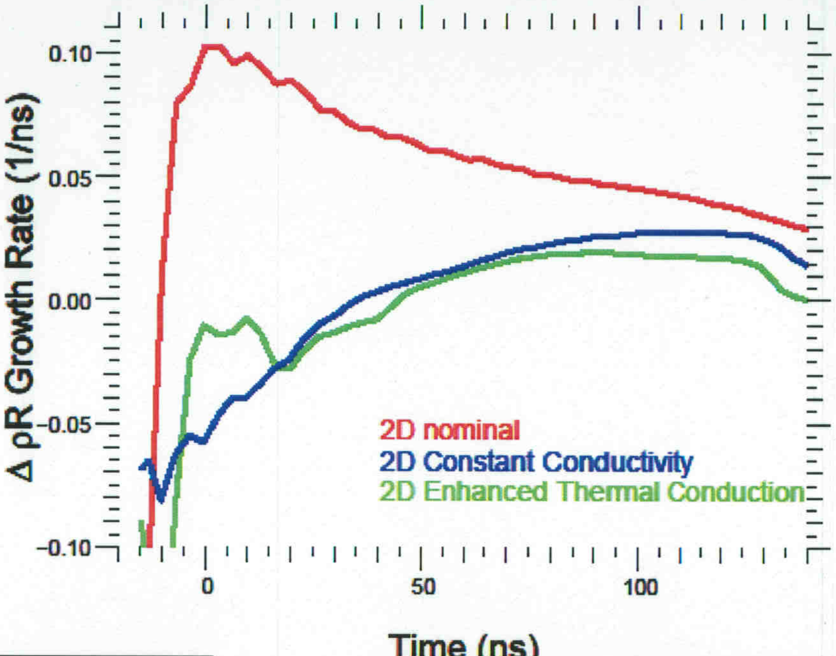


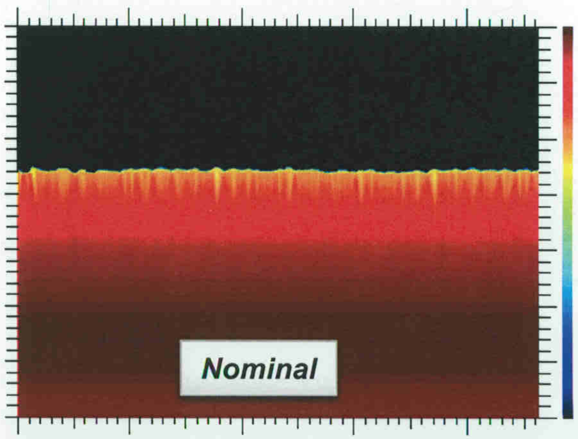


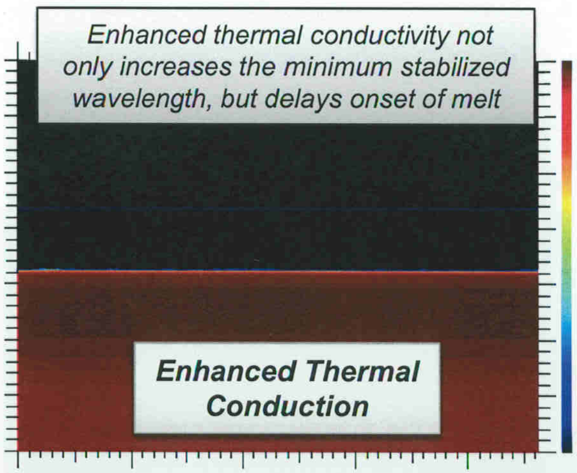
Tests with constant electrical conductivity and enhanced thermal conduction are consistent with electrothermal instabilities

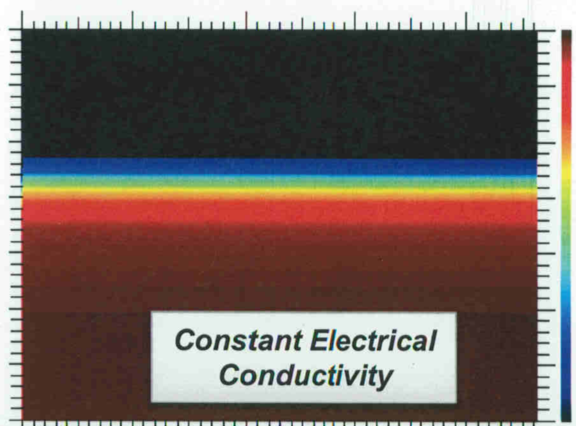
- Electrothermal instabilities should disappear if the electrical conductivity temperature dependence is removed
- Thermal conduction stabilizes short wavelengths.

$$\Gamma = \frac{j^2 \frac{\partial \eta(T)}{\partial T} - k_z^2 \kappa}{\rho c_v}$$



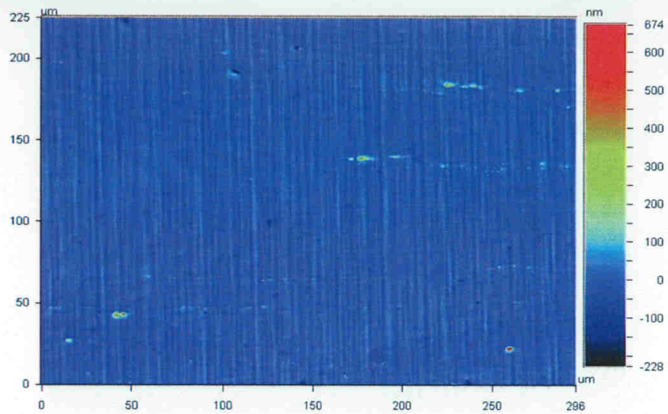




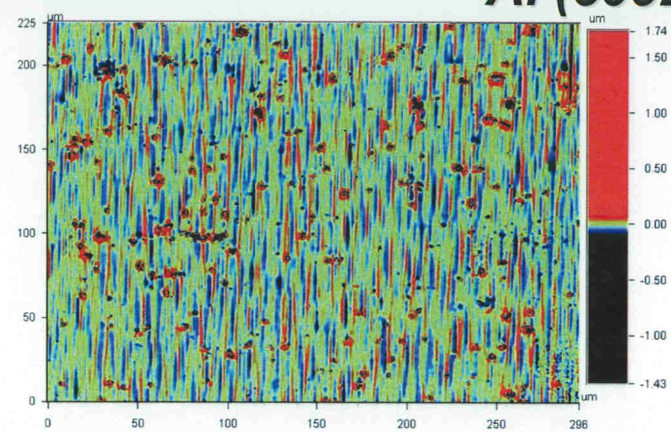


What is the most appropriate way to represent 3D surface features in a 2D code

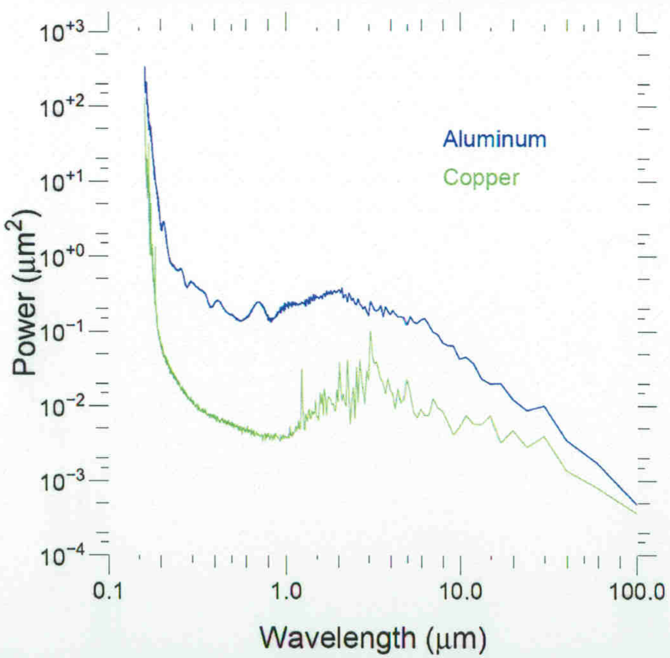
Cu



Al (5052)

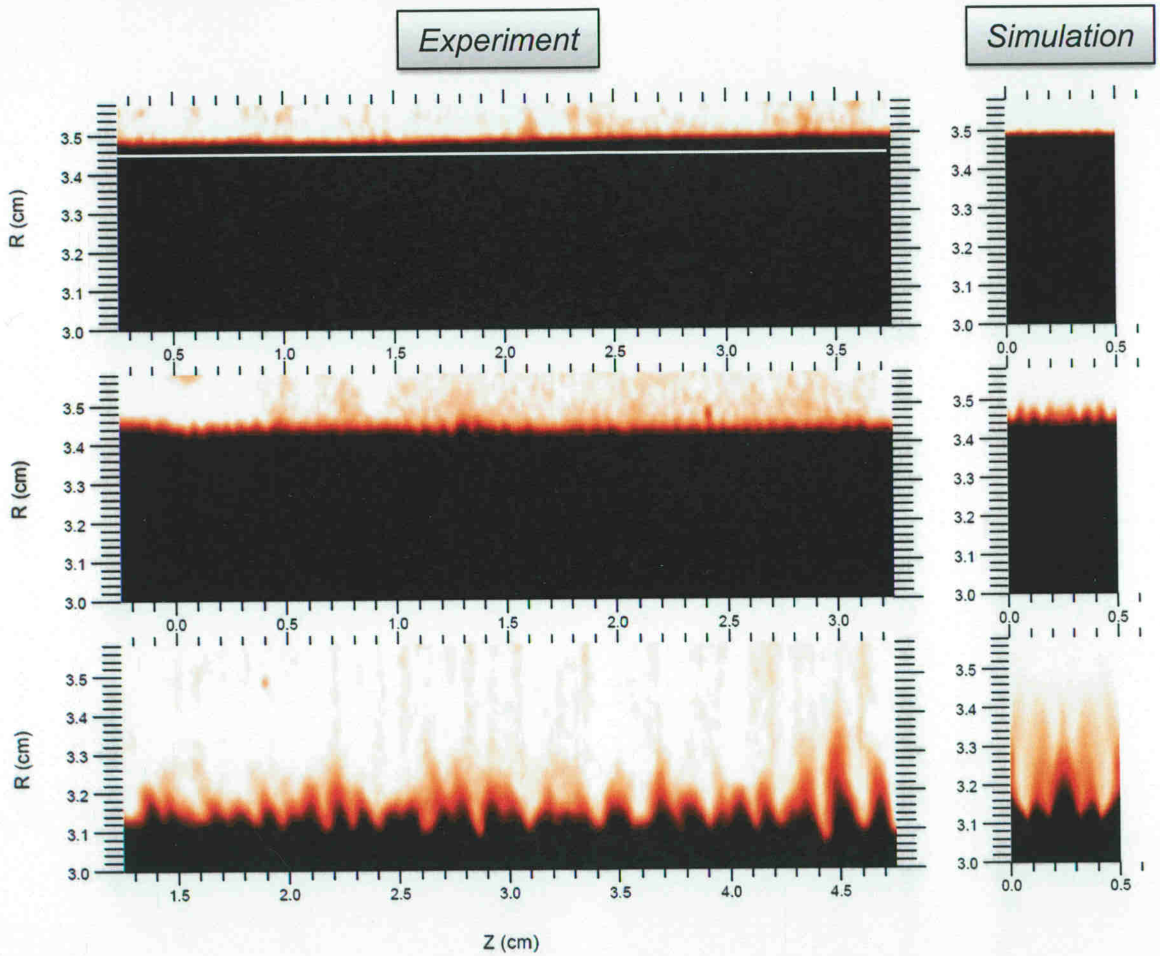


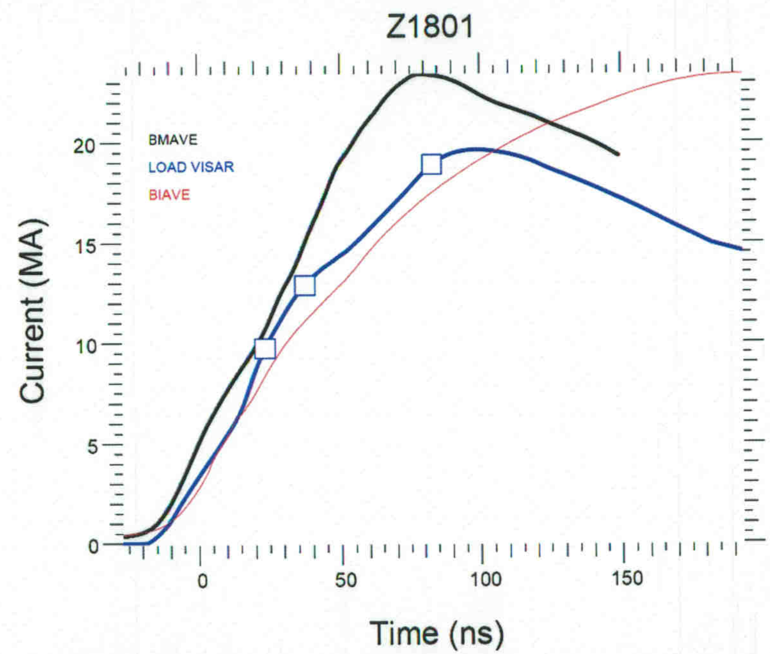
Measured Surface Roughness Spectrum



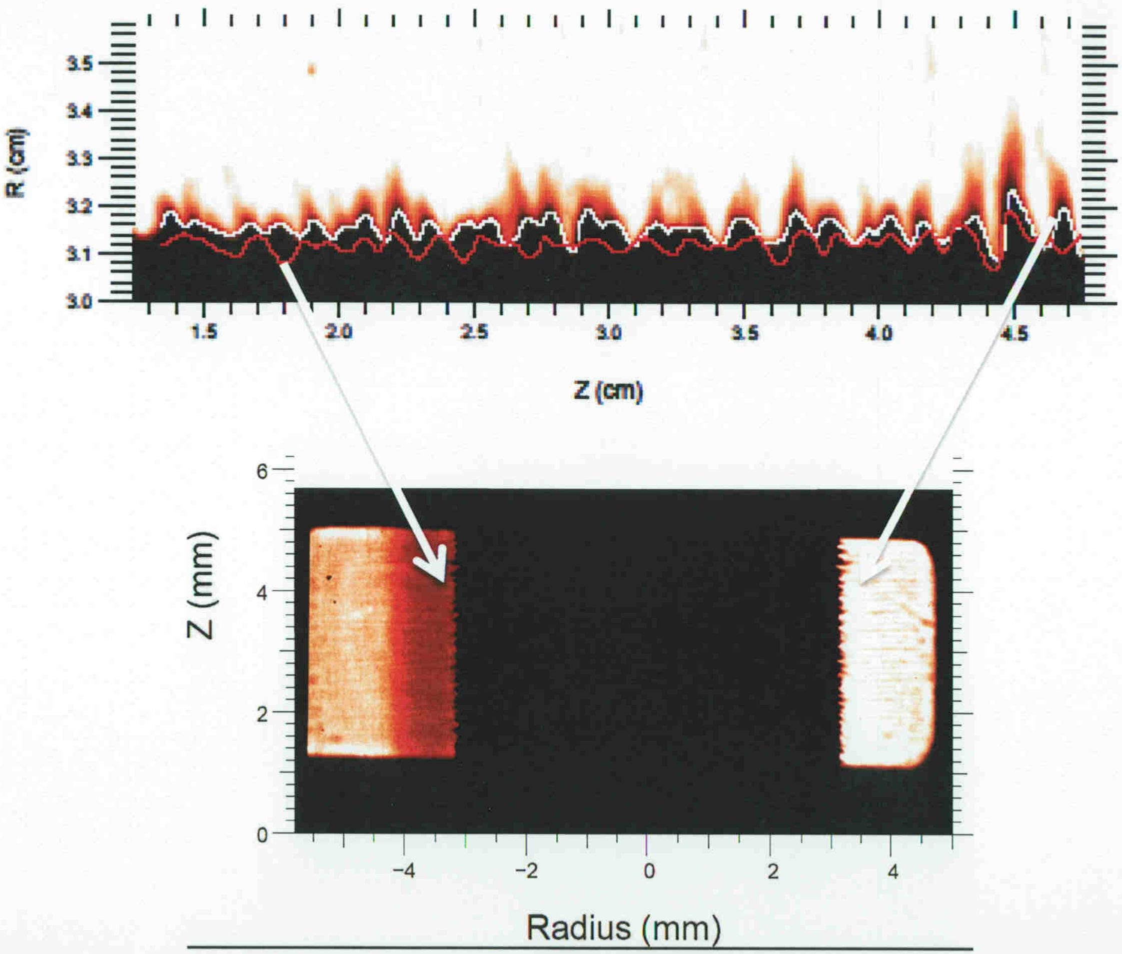
- **Do pits and large features dominate?**
- **Variety of approaches were tried**
- **Method used for simulations shown in this presentation**
  - **FFTs were taken of lineouts taken at every measured azimuthal point and then averaged**
  - **Spectrum filtered for mesh resolution**
  - **Inverse transformed onto simulation physical mesh**

Three frames of Al data were obtained which show a large range of instability growth

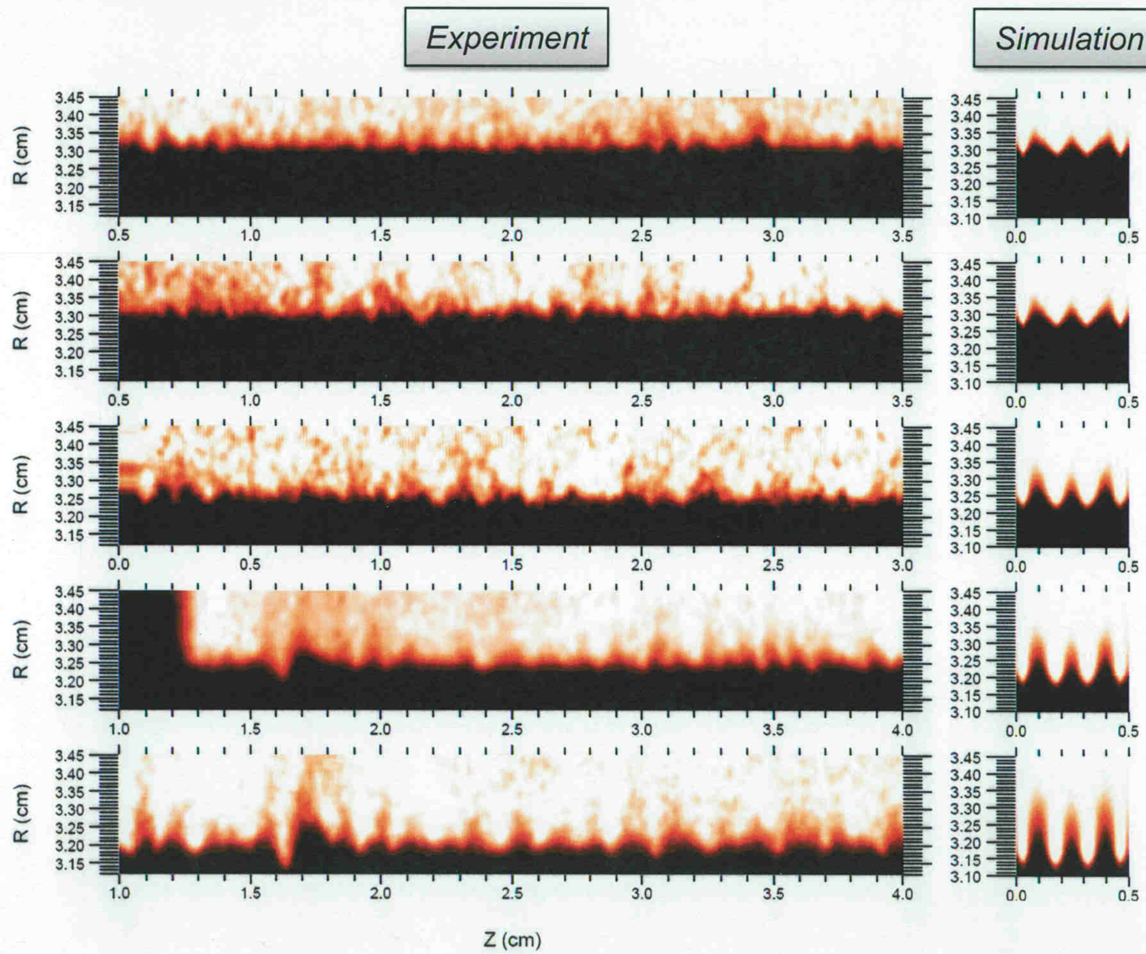


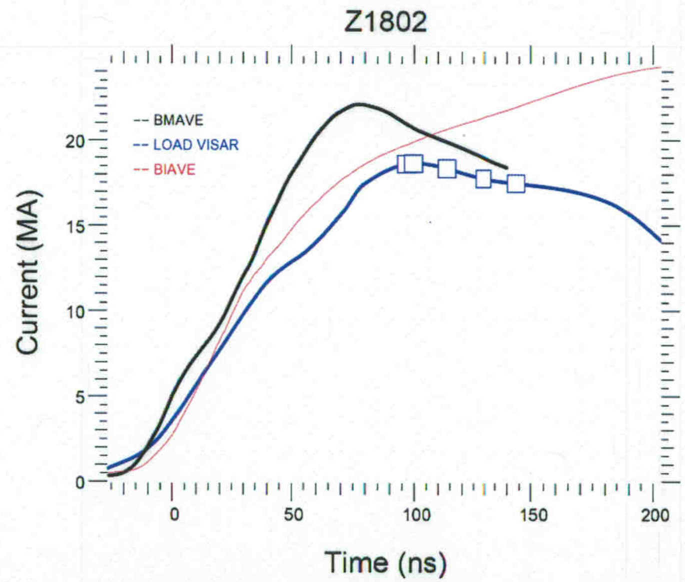


Only the largest perturbations show strong correlation with the opposite side of imaged rod



Five frames of Cu data were recorded which showed qualitative differences with simulation





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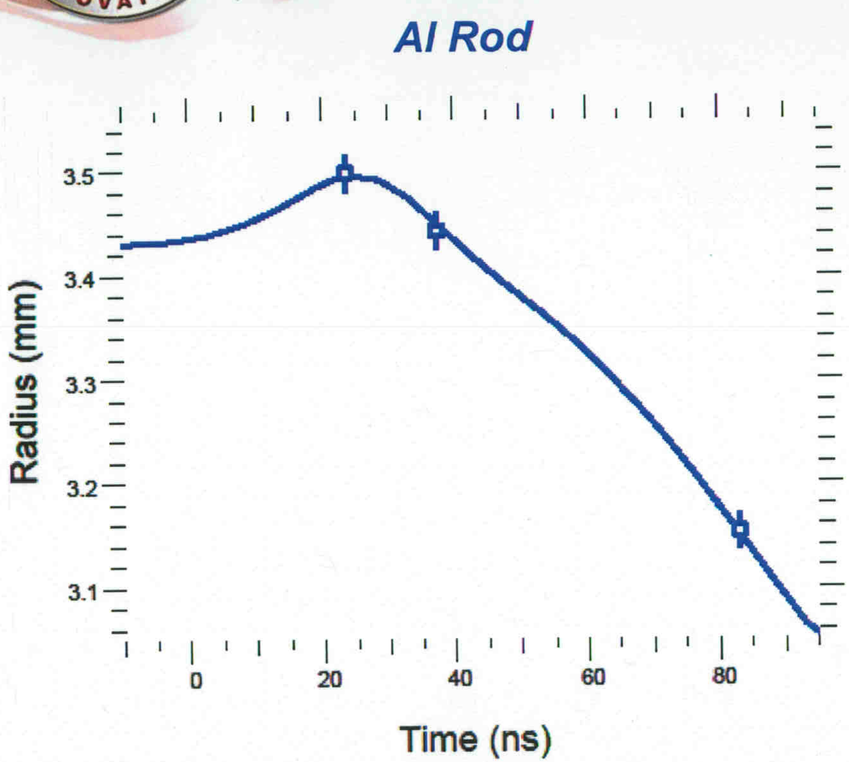
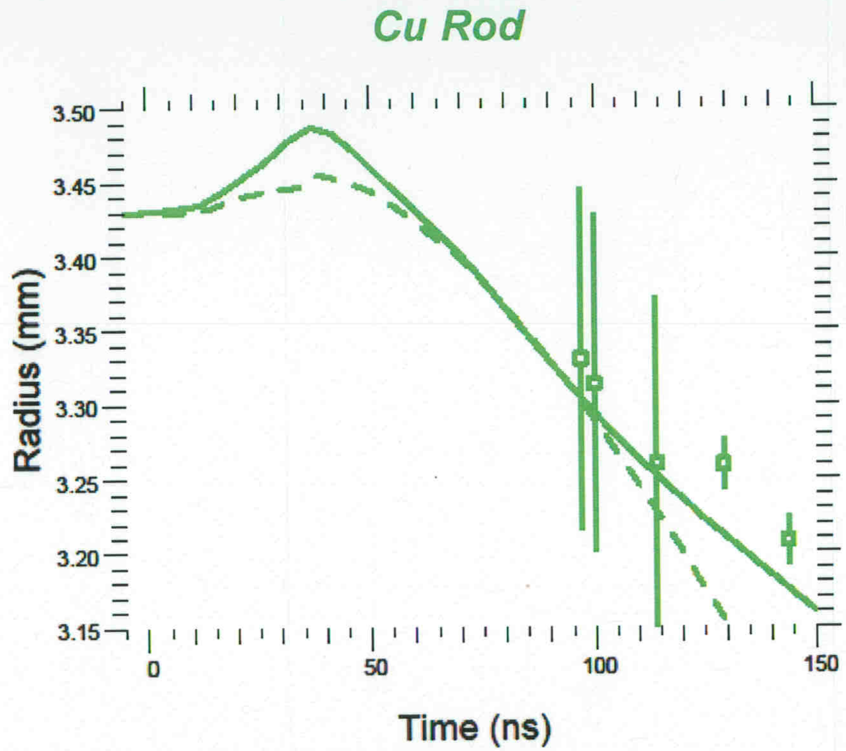
Al measurements match simulated radial trajectories exceptionally well. Cu simulations are in much poorer agreement.





- Simulated radius time history determined by calculating average radius (0.5 transmission contour) in a series of simulated radiographs

- Simulations with load Bdot current measurement (dashed curve) have even greater discrepancy

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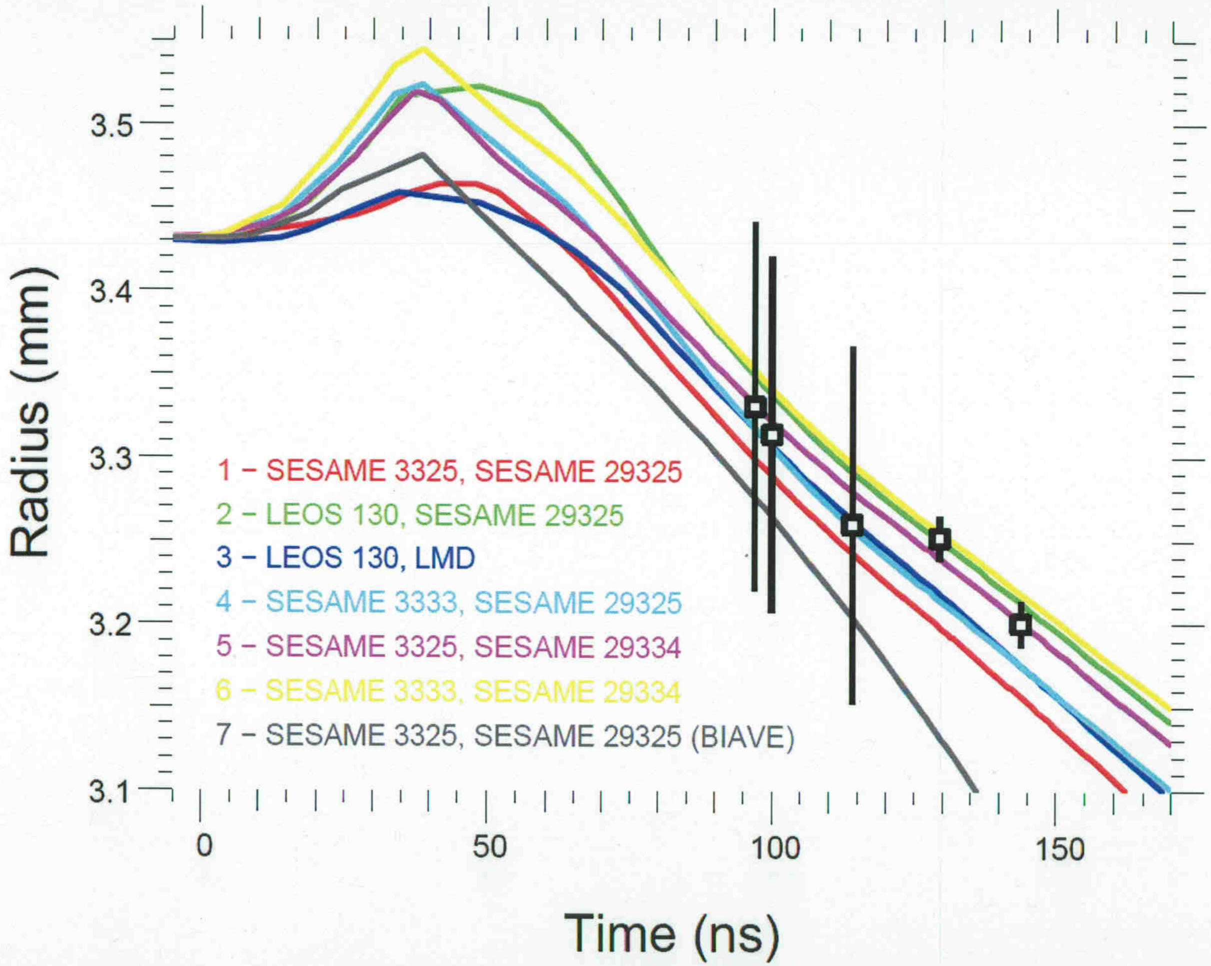
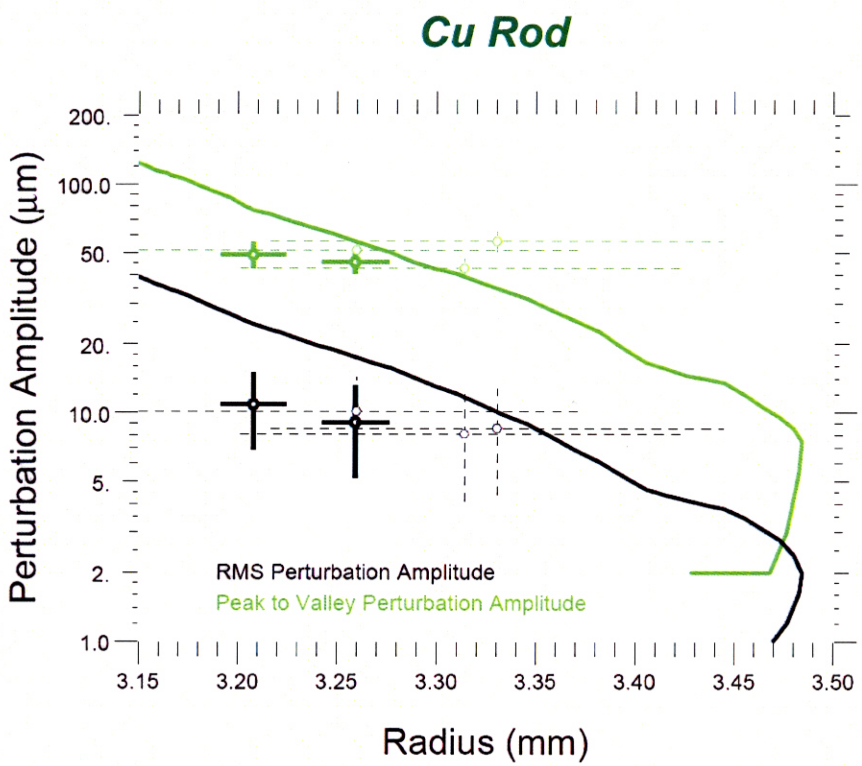
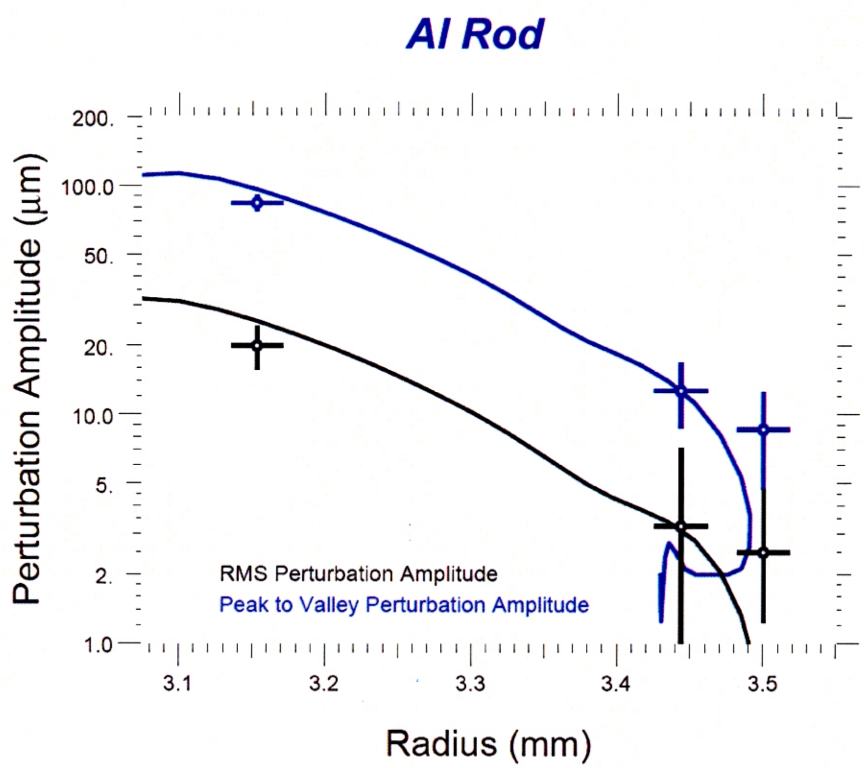
Highest fidelity copper models cannot match measured radial trajectories



Simulated Al perturbation growth lies nearly within error bars while experimental Cu growth is approximately 2X lower than simulations

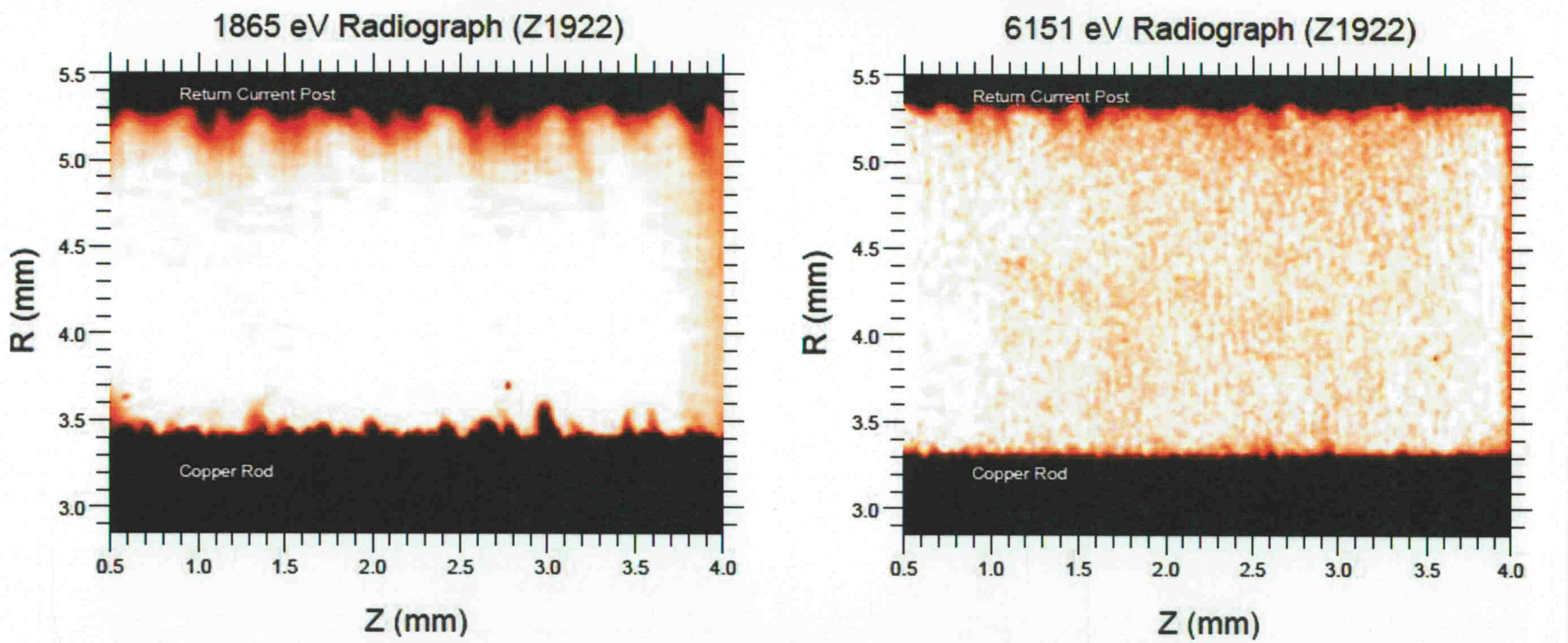




- **Majority of instability growth occurs during the surface initiation and expansion phase**

2-color monochromatic crystal backlighting showed dramatic differences in instability development

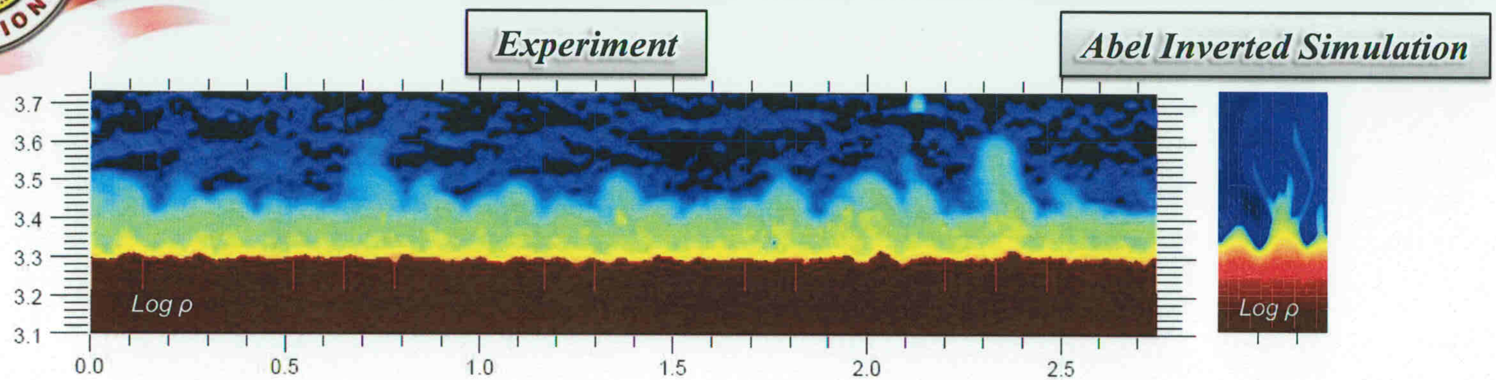
Cu has ~25 times more opacity at 1865eV than 6151eV for Te<100 eV

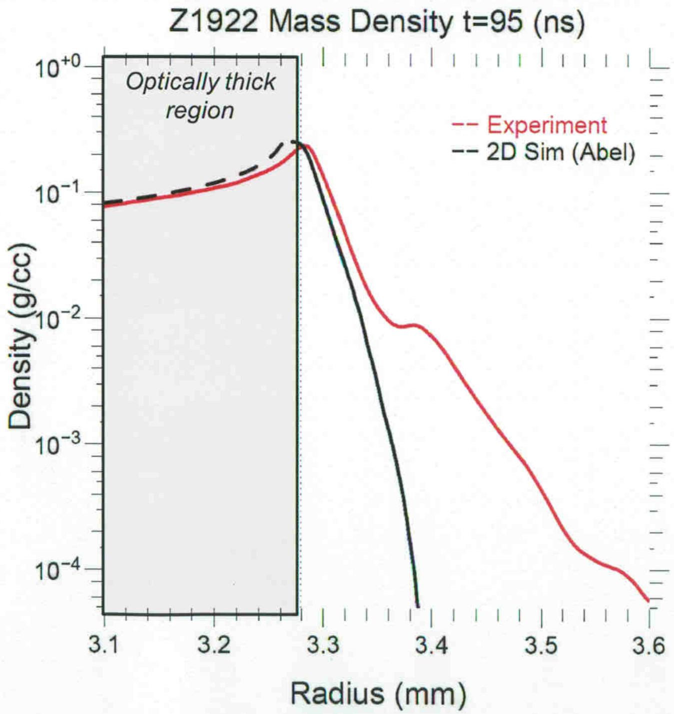


Time was chosen to be the point at which simulations predicted the maximum observable difference at 6151eV and 1865eV

2 ns interframe time

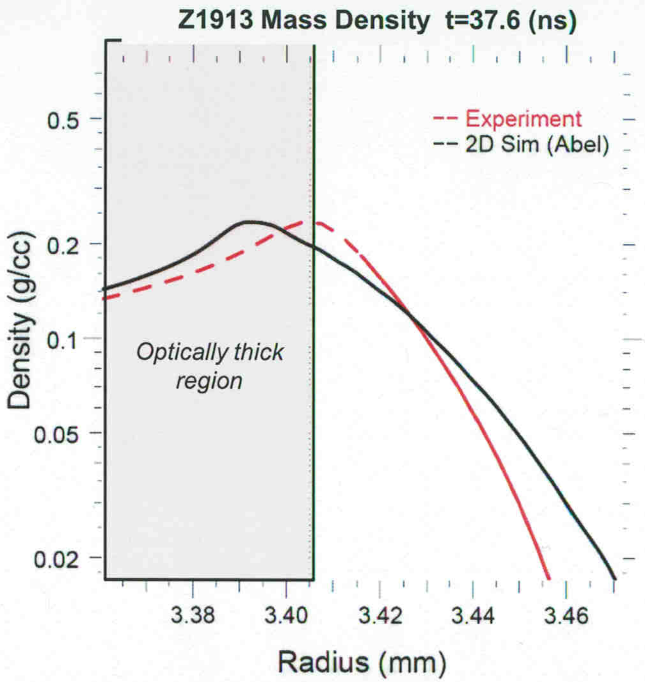
Combining the Cu 2-color images improves dynamic range and allows comparisons at much lower densities

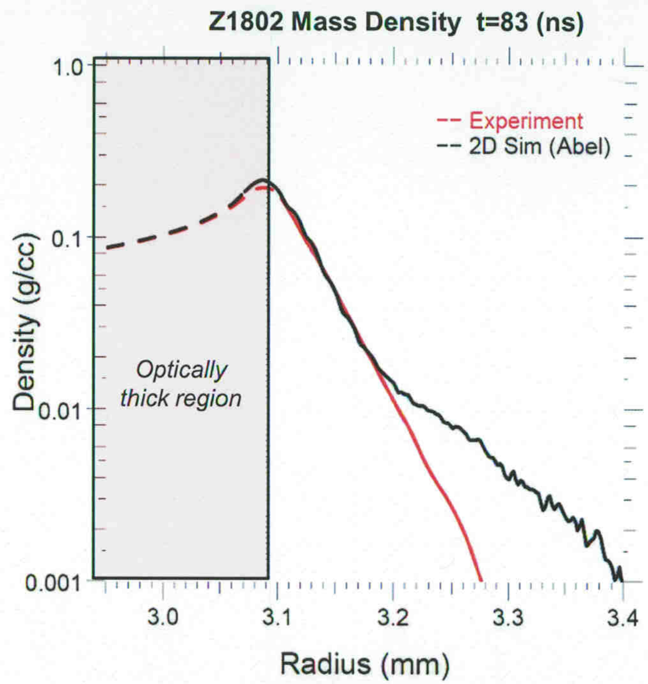




- Transmission range limited by obtained S2N of each image (5%-95%)
- 1865eV areal density range 2.0e-5 to 1.2e-3 (g/cm2)
- 6151eV areal density range 4.8e-4 to 2.8e-2 (g/cm2)
- Alignment of 2 color images
  - No preshot radiograph
  - Theoretical relative image magnification assumed
  - Radial alignment provided by shifting axially averaged areal density lineouts such that they matched in the region where they overlap
  - Axial alignment provided using a cross correlation function on the largest perturbations
  - Composite areal density map used to perform Abel inversion
- Dynamic Range of ~60 for individual images, improves to ~1400 when combined

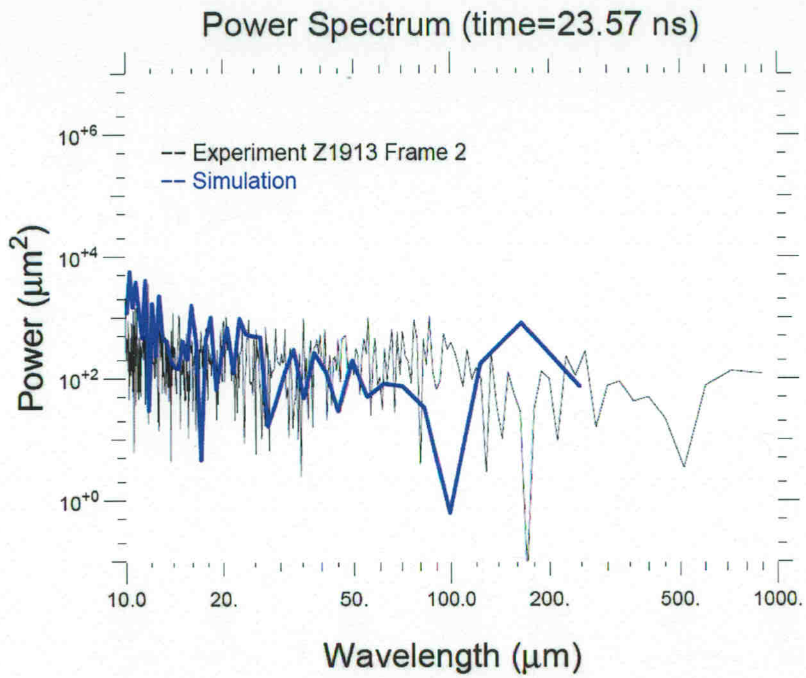
Aluminum Abel inversions show radial density profiles are in very good agreement

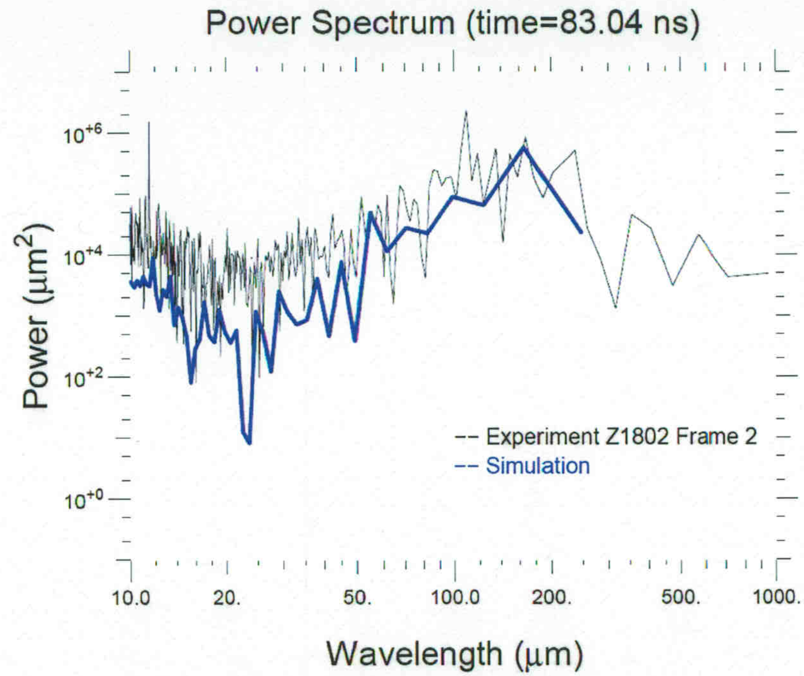




- Early time simulation may have bad better agreement if VISAR current data was recorded on that shot and used for this simulation.
- Later time agreement is excellent at densities down to 0.01 g/cc.
- Al simulations show a much broader distribution at lower densities than experiments show. This is in complete contrast to Cu which showed the opposite.

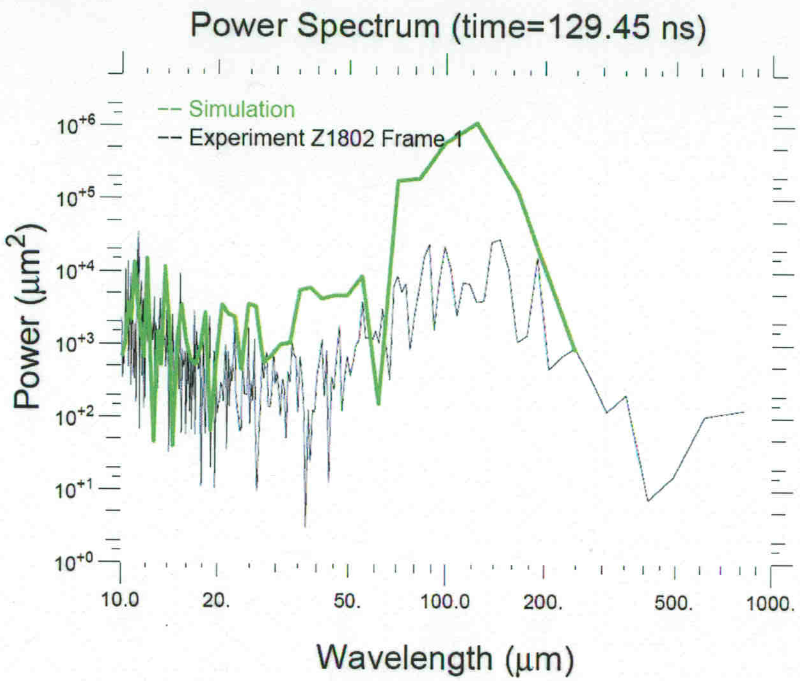
Dominant wavelengths and amplitudes agree reasonably well with AI simulations

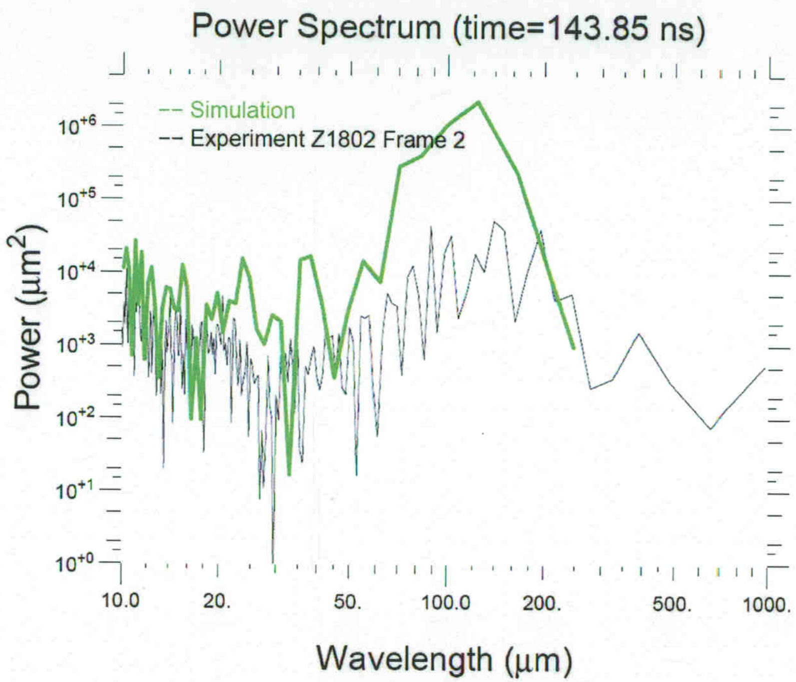




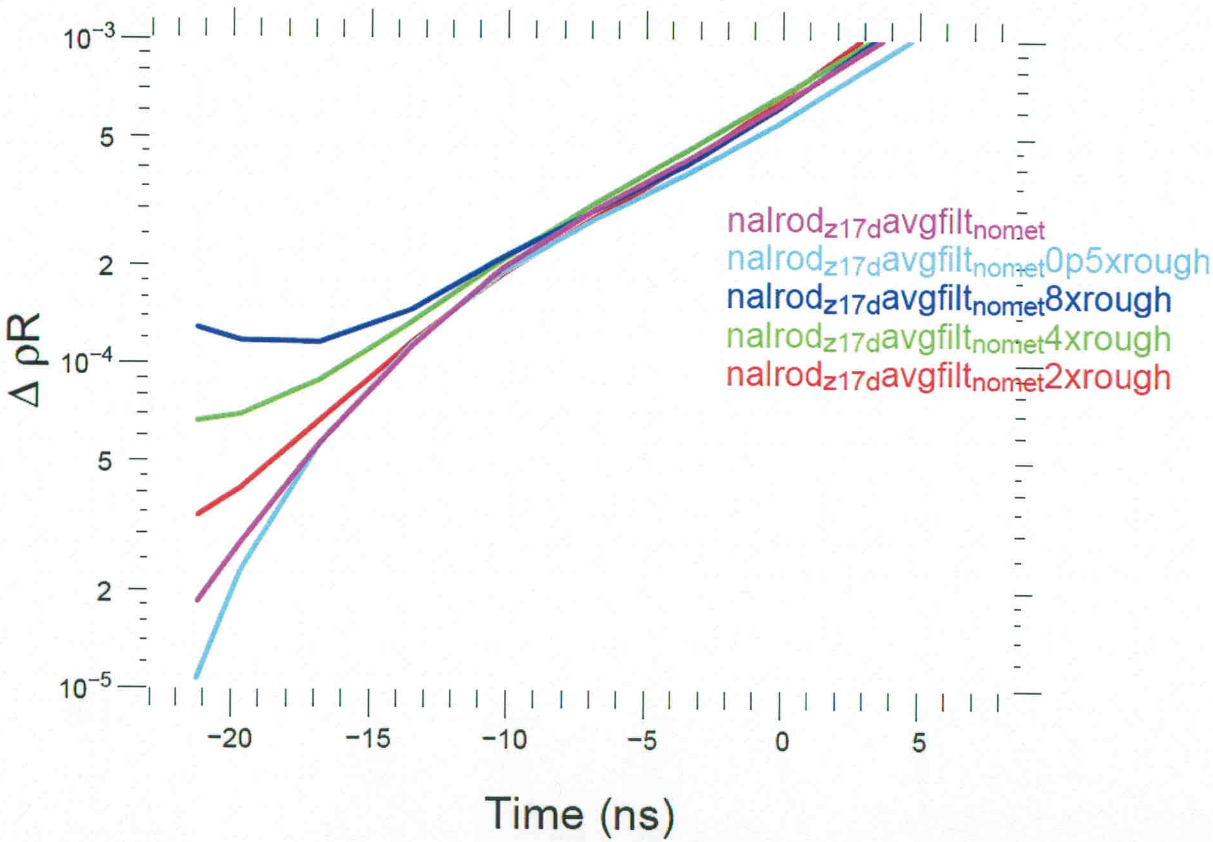
Excellent agreement is observed considering perturbations have grown 3 to 4 orders of magnitude in amplitude

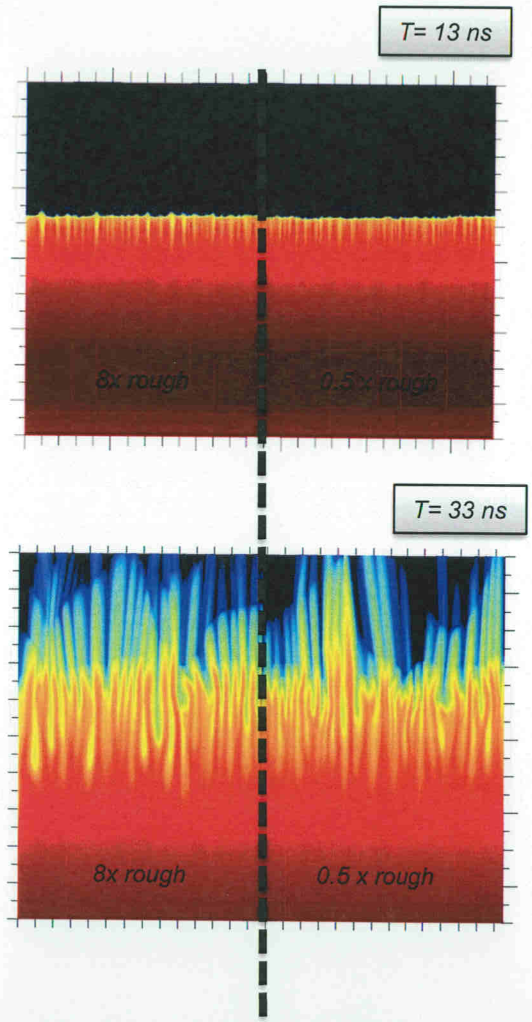
Cu simulations appear to be capturing the nonlinear development to larger wavelengths, despite having larger growth





Magnitude of calculated electrothermal instability growth is insensitive to the level of initial surface roughness





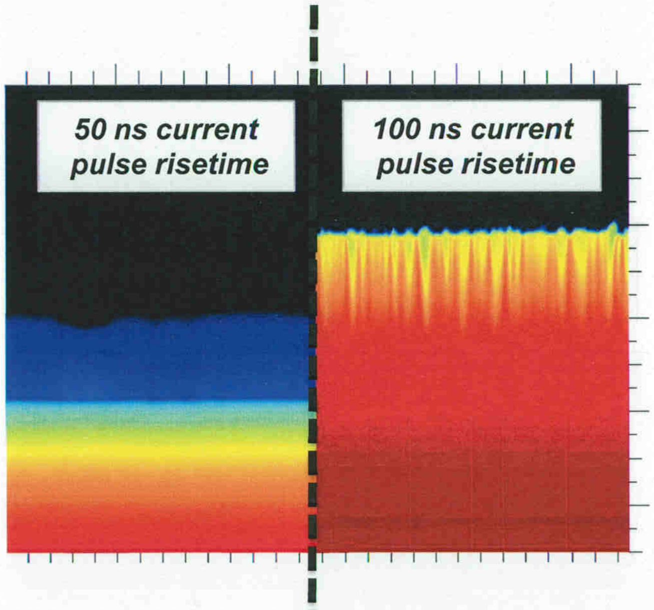
Conclusions

- Perturbation growth is consistent with hypothesis of electrothermal instabilities seeding subsequent MRT
- Solid rod experiments provide a stringent test of code predictions of instability growth from surface roughness
- Aluminum simulations match experiments extremely well but Copper material models appear to need some improvement
- Magnitude of calculated electrothermal instability growth is insensitive to the level of initial surface roughness

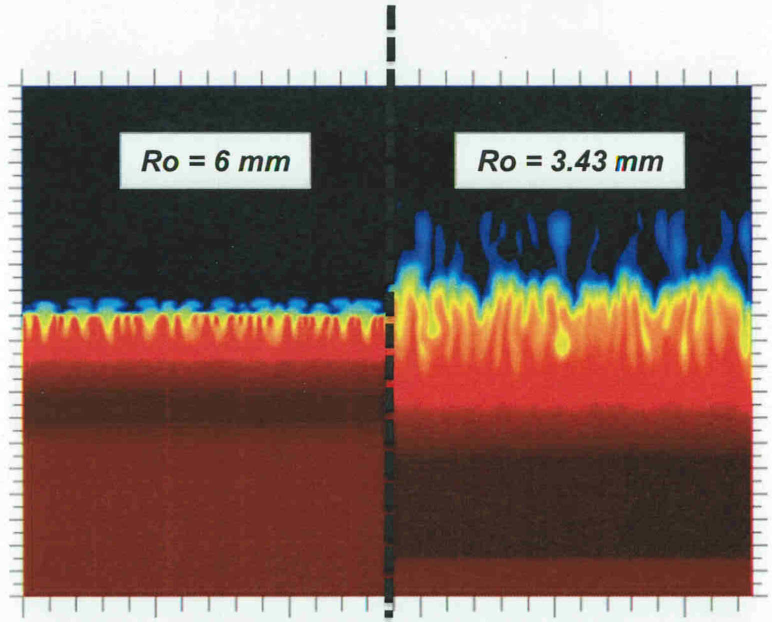
Backups

Mitigation Strategies

Reduce electrothermal growth time



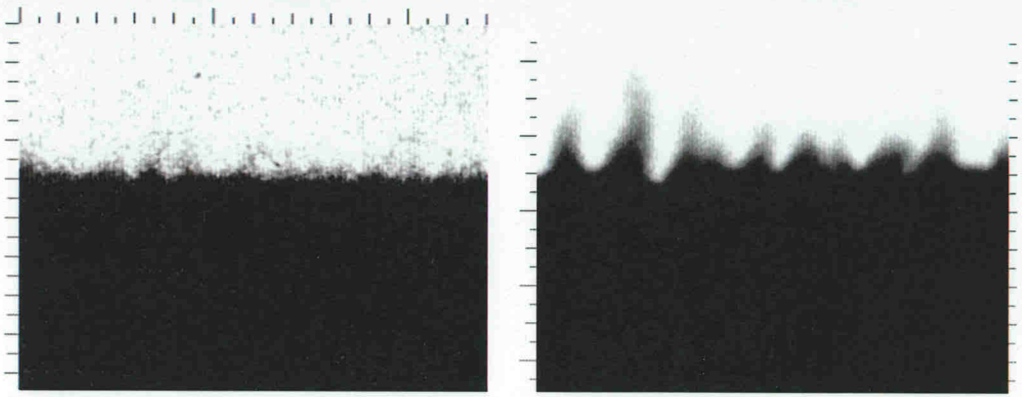
Reduce current density



$$j(r,t) \approx \frac{I(t)}{2\pi r_o \Delta(t)}$$

More instability growth is observed with Al then Cu experimentally

Cu Rod **Al Rod**



Radiographs compared at approximately same time and magnetic pressure